

Mathematica 11.3 Integration Test Results

Test results for the 29 problems in "6.6.1 $(c+dx)^m (a+b \operatorname{csch})^n m$ "

Problem 3: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int (c + d x) \operatorname{Csch}[a + b x] dx$$

Optimal (type 4, 50 leaves, 5 steps):

$$-\frac{2 (c + d x) \operatorname{ArcTanh}\left[e^{a+b x}\right]}{b} - \frac{d \operatorname{PolyLog}\left[2, -e^{a+b x}\right]}{b^2} + \frac{d \operatorname{PolyLog}\left[2, e^{a+b x}\right]}{b^2}$$

Result (type 4, 174 leaves):

$$\begin{aligned} & -\frac{c \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{a}{2}+\frac{b x}{2}\right]\right]}{b} + \frac{c \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{a}{2}+\frac{b x}{2}\right]\right]}{b} + \frac{1}{b^2} \\ & d \left(-a \operatorname{Log}\left[\operatorname{Tanh}\left[\frac{1}{2} (a + b x)\right]\right] - \frac{i}{2} (\operatorname{Log}\left[1 - e^{i (a + i b x)}\right] - \operatorname{Log}\left[1 + e^{i (a + i b x)}\right]) + \right. \\ & \quad \left. \frac{i}{2} (\operatorname{PolyLog}\left[2, -e^{i (a + i b x)}\right] - \operatorname{PolyLog}\left[2, e^{i (a + i b x)}\right]) \right) \end{aligned}$$

Problem 6: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int (c + d x)^2 \operatorname{Csch}[a + b x]^2 dx$$

Optimal (type 4, 74 leaves, 5 steps):

$$-\frac{(c + d x)^2}{b} - \frac{(c + d x)^2 \operatorname{Coth}[a + b x]}{b} + \frac{2 d (c + d x) \operatorname{Log}\left[1 - e^{2 (a + b x)}\right]}{b^2} + \frac{d^2 \operatorname{PolyLog}\left[2, e^{2 (a + b x)}\right]}{b^3}$$

Result (type 4, 277 leaves):

$$\begin{aligned}
& - \left(\left(2 c d \operatorname{Csch}[a] (-b x \operatorname{Cosh}[a] + \operatorname{Log}[\operatorname{Cosh}[b x] \operatorname{Sinh}[a] + \operatorname{Cosh}[a] \operatorname{Sinh}[b x] \operatorname{Sinh}[a]]) \right) / \right. \\
& \quad \left. \left(b^2 (-\operatorname{Cosh}[a]^2 + \operatorname{Sinh}[a]^2) \right) \right) + \frac{1}{b} \\
& \operatorname{Csch}[a] \operatorname{Csch}[a+b x] (c^2 \operatorname{Sinh}[b x] + 2 c d x \operatorname{Sinh}[b x] + d^2 x^2 \operatorname{Sinh}[b x]) + \\
& \left(d^2 \operatorname{Csch}[a] \operatorname{Sech}[a] \left(-b^2 e^{-\operatorname{ArcTanh}[\operatorname{Tanh}[a]]} x^2 + \right. \right. \\
& \quad \left. \left. \left(\frac{i}{\pi} (-b x (-\pi + 2 i \operatorname{ArcTanh}[\operatorname{Tanh}[a]])) - \pi \operatorname{Log}[1 + e^{2 b x}] - 2 (\frac{i}{\pi} b x + \frac{i}{\pi} \operatorname{ArcTanh}[\operatorname{Tanh}[a]]) \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{Log}[1 - e^{2 i (\frac{i}{\pi} b x + \frac{i}{\pi} \operatorname{ArcTanh}[\operatorname{Tanh}[a]])}] + \pi \operatorname{Log}[\operatorname{Cosh}[b x]] + 2 \frac{i}{\pi} \operatorname{ArcTanh}[\operatorname{Tanh}[a]] \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{Log}[\frac{i}{\pi} \operatorname{Sinh}[b x + \operatorname{ArcTanh}[\operatorname{Tanh}[a]]]] + \frac{i}{\pi} \operatorname{PolyLog}[2, e^{2 i (\frac{i}{\pi} b x + \frac{i}{\pi} \operatorname{ArcTanh}[\operatorname{Tanh}[a]])}] \right) \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{Tanh}[a] \right) \right/ \left(\sqrt{1 - \operatorname{Tanh}[a]^2} \right) \right) \right) / \left(b^3 \sqrt{\operatorname{Sech}[a]^2 (\operatorname{Cosh}[a]^2 - \operatorname{Sinh}[a]^2)} \right)
\end{aligned}$$

Problem 10: Result more than twice size of optimal antiderivative.

$$\int (c + d x)^2 \operatorname{Csch}[a + b x]^3 dx$$

Optimal (type 4, 154 leaves, 9 steps):

$$\begin{aligned}
& \frac{(c + d x)^2 \operatorname{ArcTanh}[e^{a+b x}]}{b} - \frac{d^2 \operatorname{ArcTanh}[\operatorname{Cosh}[a + b x]]}{b^3} - \frac{d (c + d x) \operatorname{Csch}[a + b x]}{b^2} - \\
& \frac{(c + d x)^2 \operatorname{Coth}[a + b x] \operatorname{Csch}[a + b x]}{2 b} + \frac{d (c + d x) \operatorname{PolyLog}[2, -e^{a+b x}]}{b^2} - \\
& \frac{d (c + d x) \operatorname{PolyLog}[2, e^{a+b x}]}{b^2} - \frac{d^2 \operatorname{PolyLog}[3, -e^{a+b x}]}{b^3} + \frac{d^2 \operatorname{PolyLog}[3, e^{a+b x}]}{b^3}
\end{aligned}$$

Result (type 4, 420 leaves):

$$\begin{aligned}
& - \frac{d (c + d x) \operatorname{Csch}[a]}{b^2} + \frac{(-c^2 - 2 c d x - d^2 x^2) \operatorname{Csch}[\frac{a}{2} + \frac{b x}{2}]^2}{8 b} + \\
& \frac{1}{2 b^3} (-b^2 c^2 \operatorname{Log}[1 - e^{a+b x}] + 2 d^2 \operatorname{Log}[1 - e^{a+b x}] - 2 b^2 c d x \operatorname{Log}[1 - e^{a+b x}] - \\
& b^2 d^2 x^2 \operatorname{Log}[1 - e^{a+b x}] + b^2 c^2 \operatorname{Log}[1 + e^{a+b x}] - 2 d^2 \operatorname{Log}[1 + e^{a+b x}] + \\
& 2 b^2 c d x \operatorname{Log}[1 + e^{a+b x}] + b^2 d^2 x^2 \operatorname{Log}[1 + e^{a+b x}] + 2 b d (c + d x) \operatorname{PolyLog}[2, -e^{a+b x}] - \\
& 2 b d (c + d x) \operatorname{PolyLog}[2, e^{a+b x}] - 2 d^2 \operatorname{PolyLog}[3, -e^{a+b x}] + 2 d^2 \operatorname{PolyLog}[3, e^{a+b x}]) + \\
& \frac{(-c^2 - 2 c d x - d^2 x^2) \operatorname{Sech}[\frac{a}{2} + \frac{b x}{2}]^2}{8 b} + \frac{\operatorname{Csch}[\frac{a}{2}] \operatorname{Csch}[\frac{a}{2} + \frac{b x}{2}] (c d \operatorname{Sinh}[\frac{b x}{2}] + d^2 x \operatorname{Sinh}[\frac{b x}{2}])}{2 b^2} + \\
& \frac{\operatorname{Sech}[\frac{a}{2}] \operatorname{Sech}[\frac{a}{2} + \frac{b x}{2}] (c d \operatorname{Sinh}[\frac{b x}{2}] + d^2 x \operatorname{Sinh}[\frac{b x}{2}])}{2 b^2}
\end{aligned}$$

Problem 11: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int (c + d x) \operatorname{Csch}[a + b x]^3 dx$$

Optimal (type 4, 92 leaves, 6 steps):

$$\frac{(c+d x) \operatorname{ArcTanh}[e^{a+b x}]}{b} - \frac{d \operatorname{Csch}[a+b x]}{2 b^2} - \frac{(c+d x) \operatorname{Coth}[a+b x] \operatorname{Csch}[a+b x]}{2 b} + \frac{d \operatorname{PolyLog}[2, -e^{a+b x}]}{2 b^2} - \frac{d \operatorname{PolyLog}[2, e^{a+b x}]}{2 b^2}$$

Result (type 4, 332 leaves):

$$\begin{aligned} & -\frac{d x \operatorname{Csch}\left[\frac{a}{2}+\frac{b x}{2}\right]^2}{8 b}-\frac{c \operatorname{Csch}\left[\frac{1}{2}(a+b x)\right]^2}{8 b}+\frac{c \operatorname{Log}[\operatorname{Cosh}\left[\frac{1}{2}(a+b x)\right]]}{2 b}-\frac{c \operatorname{Log}[\operatorname{Sinh}\left[\frac{1}{2}(a+b x)\right]]}{2 b}- \\ & \frac{\frac{1}{2} d \left(-a \operatorname{Log}[\operatorname{Tanh}\left[\frac{1}{2}(a+b x)\right]]-\frac{i}{2}((\frac{i}{2} a+\frac{i}{2} b x)(\operatorname{Log}[1-e^{\frac{i}{2}(\frac{i}{2} a+\frac{i}{2} b x)}]-\operatorname{Log}[1+e^{\frac{i}{2}(\frac{i}{2} a+\frac{i}{2} b x)}])\right. \\ & \left.\left.i\left(\operatorname{PolyLog}[2,-e^{\frac{i}{2}(\frac{i}{2} a+\frac{i}{2} b x)}]-\operatorname{PolyLog}[2,e^{\frac{i}{2}(\frac{i}{2} a+\frac{i}{2} b x)}]\right)\right)- \\ & \frac{d x \operatorname{Sech}\left[\frac{a}{2}+\frac{b x}{2}\right]^2}{8 b}-\frac{c \operatorname{Sech}\left[\frac{1}{2}(a+b x)\right]^2}{8 b}+\frac{d \operatorname{Csch}\left[\frac{a}{2}\right] \operatorname{Csch}\left[\frac{a}{2}+\frac{b x}{2}\right] \operatorname{Sinh}\left[\frac{b x}{2}\right]}{4 b^2}+ \\ & \frac{d \operatorname{Sech}\left[\frac{a}{2}\right] \operatorname{Sech}\left[\frac{a}{2}+\frac{b x}{2}\right] \operatorname{Sinh}\left[\frac{b x}{2}\right]}{4 b^2} \end{aligned}$$

Problem 17: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(e+f x)^3 \operatorname{Cosh}[c+d x]}{a+b \operatorname{Csch}[c+d x]} dx$$

Optimal (type 4, 448 leaves, 17 steps):

$$\begin{aligned} & \frac{b (e+f x)^4}{4 a^2 f}-\frac{6 f^3 \operatorname{Cosh}[c+d x]}{a d^4}-\frac{3 f (e+f x)^2 \operatorname{Cosh}[c+d x]}{a d^2}-\frac{b (e+f x)^3 \operatorname{Log}\left[1+\frac{a e^{c+d x}}{b-\sqrt{a^2+b^2}}\right]}{a^2 d}- \\ & \frac{b (e+f x)^3 \operatorname{Log}\left[1+\frac{a e^{c+d x}}{b+\sqrt{a^2+b^2}}\right]}{a^2 d}-\frac{3 b f (e+f x)^2 \operatorname{PolyLog}\left[2,-\frac{a e^{c+d x}}{b-\sqrt{a^2+b^2}}\right]}{a^2 d^2}- \\ & \frac{3 b f (e+f x)^2 \operatorname{PolyLog}\left[2,-\frac{a e^{c+d x}}{b+\sqrt{a^2+b^2}}\right]}{a^2 d^2}+\frac{6 b f^2 (e+f x) \operatorname{PolyLog}\left[3,-\frac{a e^{c+d x}}{b-\sqrt{a^2+b^2}}\right]}{a^2 d^3}+ \\ & \frac{6 b f^2 (e+f x) \operatorname{PolyLog}\left[3,-\frac{a e^{c+d x}}{b+\sqrt{a^2+b^2}}\right]}{a^2 d^3}-\frac{6 b f^3 \operatorname{PolyLog}\left[4,-\frac{a e^{c+d x}}{b-\sqrt{a^2+b^2}}\right]}{a^2 d^4}- \\ & \frac{6 b f^3 \operatorname{PolyLog}\left[4,-\frac{a e^{c+d x}}{b+\sqrt{a^2+b^2}}\right]}{a^2 d^4}+\frac{6 f^2 (e+f x) \operatorname{Sinh}[c+d x]}{a d^3}+\frac{(e+f x)^3 \operatorname{Sinh}[c+d x]}{a d} \end{aligned}$$

Result (type 4, 1635 leaves):

$$\frac{1}{2 a^2 d^3 (a+b \operatorname{Csch}[c+d x])} e f^2 \operatorname{Csch}[c+d x]$$

$$\begin{aligned}
& \left(-12 b d x \operatorname{PolyLog}[2, -\frac{a e^{2 c+d x}}{b e^c - \sqrt{(a^2 + b^2) e^{2 c}}}] - 12 b d x \operatorname{PolyLog}[2, -\frac{a e^{2 c+d x}}{b e^c + \sqrt{(a^2 + b^2) e^{2 c}}}] + \right. \\
& \quad e^{-c} \left(2 b d^3 e^c x^3 - 6 a \operatorname{Cosh}[d x] + 6 a e^{2 c} \operatorname{Cosh}[d x] - 6 a d x \operatorname{Cosh}[d x] - 6 a d e^{2 c} x \operatorname{Cosh}[d x] - \right. \\
& \quad 3 a d^2 x^2 \operatorname{Cosh}[d x] + 3 a d^2 e^{2 c} x^2 \operatorname{Cosh}[d x] - 6 b d^2 e^c x^2 \operatorname{Log}\left[1 + \frac{a e^{2 c+d x}}{b e^c - \sqrt{(a^2 + b^2) e^{2 c}}}\right] - \\
& \quad 6 b d^2 e^c x^2 \operatorname{Log}\left[1 + \frac{a e^{2 c+d x}}{b e^c + \sqrt{(a^2 + b^2) e^{2 c}}}\right] + 12 b e^c \operatorname{PolyLog}[3, -\frac{a e^{2 c+d x}}{b e^c - \sqrt{(a^2 + b^2) e^{2 c}}}] + \\
& \quad 12 b e^c \operatorname{PolyLog}[3, -\frac{a e^{2 c+d x}}{b e^c + \sqrt{(a^2 + b^2) e^{2 c}}}] + 6 a \operatorname{Sinh}[d x] + 6 a e^{2 c} \operatorname{Sinh}[d x] + \\
& \quad 6 a d x \operatorname{Sinh}[d x] - 6 a d e^{2 c} x \operatorname{Sinh}[d x] + 3 a d^2 x^2 \operatorname{Sinh}[d x] + 3 a d^2 e^{2 c} x^2 \operatorname{Sinh}[d x] \Big) \\
& \left(b + a \operatorname{Sinh}[c + d x] \right) + \frac{1}{4 a^2 d^4 (a + b \operatorname{Csch}[c + d x])} f^3 \operatorname{Csch}[c + d x] \\
& \left(-12 b d^2 x^2 \operatorname{PolyLog}[2, -\frac{a e^{2 c+d x}}{b e^c - \sqrt{(a^2 + b^2) e^{2 c}}}] + \right. \\
& \quad e^{-c} \left(b d^4 e^c x^4 - 12 a \operatorname{Cosh}[d x] - 12 a e^{2 c} \operatorname{Cosh}[d x] - 12 a d x \operatorname{Cosh}[d x] + 12 a d e^{2 c} x \operatorname{Cosh}[d x] - \right. \\
& \quad 6 a d^2 x^2 \operatorname{Cosh}[d x] - 6 a d^2 e^{2 c} x^2 \operatorname{Cosh}[d x] - 2 a d^3 x^3 \operatorname{Cosh}[d x] + 2 a d^3 e^{2 c} x^3 \operatorname{Cosh}[d x] - \\
& \quad 4 b d^3 e^c x^3 \operatorname{Log}\left[1 + \frac{a e^{2 c+d x}}{b e^c - \sqrt{(a^2 + b^2) e^{2 c}}}\right] - 4 b d^3 e^c x^3 \operatorname{Log}\left[1 + \frac{a e^{2 c+d x}}{b e^c + \sqrt{(a^2 + b^2) e^{2 c}}}\right] - \\
& \quad 12 b d^2 e^c x^2 \operatorname{PolyLog}[2, -\frac{a e^{2 c+d x}}{b e^c + \sqrt{(a^2 + b^2) e^{2 c}}}] + 24 b d e^c x \\
& \quad \operatorname{PolyLog}[3, -\frac{a e^{2 c+d x}}{b e^c - \sqrt{(a^2 + b^2) e^{2 c}}}] + 24 b d e^c x \operatorname{PolyLog}[3, -\frac{a e^{2 c+d x}}{b e^c + \sqrt{(a^2 + b^2) e^{2 c}}}] - \\
& \quad 24 b e^c \operatorname{PolyLog}[4, -\frac{a e^{2 c+d x}}{b e^c - \sqrt{(a^2 + b^2) e^{2 c}}}] - 24 b e^c \operatorname{PolyLog}[4, -\frac{a e^{2 c+d x}}{b e^c + \sqrt{(a^2 + b^2) e^{2 c}}}] + \\
& \quad 12 a \operatorname{Sinh}[d x] - 12 a e^{2 c} \operatorname{Sinh}[d x] + 12 a d x \operatorname{Sinh}[d x] + 12 a d e^{2 c} x \operatorname{Sinh}[d x] + \\
& \quad 6 a d^2 x^2 \operatorname{Sinh}[d x] - 6 a d^2 e^{2 c} x^2 \operatorname{Sinh}[d x] + 2 a d^3 x^3 \operatorname{Sinh}[d x] + 2 a d^3 e^{2 c} x^3 \operatorname{Sinh}[d x] \Big) \\
& \left(b + a \operatorname{Sinh}[c + d x] \right) + \left(e^3 \operatorname{Csch}[c + d x] (b + a \operatorname{Sinh}[c + d x]) \right. \\
& \quad \left. \left(-\frac{2 b \operatorname{Log}[b + a \operatorname{Sinh}[c + d x]]}{a^2 d} + \frac{2 \operatorname{Sinh}[c + d x]}{a d} \right) \right) /
\end{aligned}$$

$$\begin{aligned}
& \left(2 \left(a + b \operatorname{Csch}[c + d x] \right) \right) + \frac{1}{a^2 d^2 \left(a + b \operatorname{Csch}[c + d x] \right)} \\
& 3 \\
& e^2 \\
& f \\
& \operatorname{Csch}[c + d x] \\
& \left(b + a \operatorname{Sinh}[c + d x] \right) \\
& \left(-a \operatorname{Cosh}[c + d x] - b (c + d x) \operatorname{Log}[b + a \operatorname{Sinh}[c + d x]] + \right. \\
& \left. b c \operatorname{Log}\left[1 + \frac{a \operatorname{Sinh}[c + d x]}{b}\right] + \frac{1}{8} b \left(2 c + \frac{i}{2} \pi + 2 d x \right)^2 - \right. \\
& \left. 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i b}{a}}}{\sqrt{2}}\right] \operatorname{ArcTan}\left[\frac{(i a + b) \operatorname{Cot}\left[\frac{1}{4} (2 i c + \pi + 2 i d x)\right]}{\sqrt{a^2 + b^2}}\right] - \right. \\
& \left. \frac{1}{2} \left(-2 i c + \pi - 2 i d x + 4 \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i b}{a}}}{\sqrt{2}}\right] \right) \operatorname{Log}\left[1 + \frac{(-b + \sqrt{a^2 + b^2}) e^{c+d x}}{a}\right] - \right. \\
& \left. \frac{1}{2} \left(-2 i c + \pi - 2 i d x - 4 \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i b}{a}}}{\sqrt{2}}\right] \right) \operatorname{Log}\left[1 - \frac{(b + \sqrt{a^2 + b^2}) e^{c+d x}}{a}\right] + \right. \\
& \left. \left(\frac{\pi}{2} - i (c + d x) \right) \operatorname{Log}[b + a \operatorname{Sinh}[c + d x]] + i \left(\operatorname{PolyLog}\left[2, \frac{(b - \sqrt{a^2 + b^2}) e^{c+d x}}{a}\right] + \right. \right. \\
& \left. \left. \operatorname{PolyLog}\left[2, \frac{(b + \sqrt{a^2 + b^2}) e^{c+d x}}{a}\right] \right) \right) + a d x \operatorname{Sinh}[c + d x]
\end{aligned}$$

Problem 18: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(e+f x)^2 \cosh[c+d x]}{a+b \operatorname{Csch}[c+d x]} dx$$

Optimal (type 4, 330 leaves, 14 steps):

$$\begin{aligned} & \frac{b (e+f x)^3}{3 a^2 f} - \frac{2 f (e+f x) \cosh[c+d x]}{a d^2} - \frac{b (e+f x)^2 \log[1 + \frac{a e^{c+d x}}{b - \sqrt{a^2+b^2}}]}{a^2 d} - \\ & \frac{b (e+f x)^2 \log[1 + \frac{a e^{c+d x}}{b + \sqrt{a^2+b^2}}]}{a^2 d} - \frac{2 b f (e+f x) \operatorname{PolyLog}[2, -\frac{a e^{c+d x}}{b - \sqrt{a^2+b^2}}]}{a^2 d^2} - \\ & \frac{2 b f (e+f x) \operatorname{PolyLog}[2, -\frac{a e^{c+d x}}{b + \sqrt{a^2+b^2}}]}{a^2 d^2} + \frac{2 b f^2 \operatorname{PolyLog}[3, -\frac{a e^{c+d x}}{b - \sqrt{a^2+b^2}}]}{a^2 d^3} + \\ & \frac{2 b f^2 \operatorname{PolyLog}[3, -\frac{a e^{c+d x}}{b + \sqrt{a^2+b^2}}]}{a^2 d^3} + \frac{2 f^2 \sinh[c+d x]}{a d^3} + \frac{(e+f x)^2 \sinh[c+d x]}{a d} \end{aligned}$$

Result (type 4, 971 leaves):

$$\begin{aligned} & \frac{1}{6 a^2 d^3 (a+b \operatorname{Csch}[c+d x])} - f^2 \operatorname{Csch}[c+d x] \\ & \left(-12 b d x \operatorname{PolyLog}[2, -\frac{a e^{2 c+d x}}{b e^c - \sqrt{(a^2+b^2) e^{2 c}}}] - 12 b d x \operatorname{PolyLog}[2, -\frac{a e^{2 c+d x}}{b e^c + \sqrt{(a^2+b^2) e^{2 c}}}] + \right. \\ & e^{-c} \left(2 b d^3 e^c x^3 - 6 a \cosh[d x] + 6 a e^{2 c} \cosh[d x] - 6 a d x \cosh[d x] - 6 a d e^{2 c} x \cosh[d x] - \right. \\ & 3 a d^2 x^2 \cosh[d x] + 3 a d^2 e^{2 c} x^2 \cosh[d x] - 6 b d^2 e^c x^2 \log[1 + \frac{a e^{2 c+d x}}{b e^c - \sqrt{(a^2+b^2) e^{2 c}}}] - \\ & 6 b d^2 e^c x^2 \log[1 + \frac{a e^{2 c+d x}}{b e^c + \sqrt{(a^2+b^2) e^{2 c}}}] + 12 b e^c \operatorname{PolyLog}[3, -\frac{a e^{2 c+d x}}{b e^c - \sqrt{(a^2+b^2) e^{2 c}}}] + \\ & 12 b e^c \operatorname{PolyLog}[3, -\frac{a e^{2 c+d x}}{b e^c + \sqrt{(a^2+b^2) e^{2 c}}}] + 6 a \sinh[d x] + 6 a e^{2 c} \sinh[d x] + \\ & 6 a d x \sinh[d x] - 6 a d e^{2 c} x \sinh[d x] + 3 a d^2 x^2 \sinh[d x] + 3 a d^2 e^{2 c} x^2 \sinh[d x] \Big) \\ & (b + a \sinh[c+d x]) + \left(e^2 \operatorname{Csch}[c+d x] (b + a \sinh[c+d x]) \right. \\ & \left(-\frac{2 b \log[b + a \sinh[c+d x]]}{a^2 d} + \frac{2 \sinh[c+d x]}{a d} \right) \Big) / \\ & (2 (a + b \operatorname{Csch}[c+d x])) + \frac{1}{a^2 d^2 (a+b \operatorname{Csch}[c+d x])} \\ & 2 e f \operatorname{Csch}[c+d x] \\ & (b + a \sinh[c+d x]) \end{aligned}$$

$$\begin{aligned}
& \left(-a \operatorname{Cosh}[c + d x] - b (c + d x) \operatorname{Log}[b + a \operatorname{Sinh}[c + d x]] + \right. \\
& b c \operatorname{Log}\left[1 + \frac{a \operatorname{Sinh}[c + d x]}{b}\right] + \frac{i}{8} b \left(-\frac{1}{8} i (2 c + i \pi + 2 d x)^2 - \right. \\
& 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i b}{a}}}{\sqrt{2}}\right] \operatorname{ArcTan}\left[\frac{(i a + b) \operatorname{Cot}\left[\frac{1}{4} (2 i c + \pi + 2 i d x)\right]}{\sqrt{a^2 + b^2}}\right] - \\
& \frac{1}{2} \left(-2 i c + \pi - 2 i d x + 4 \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i b}{a}}}{\sqrt{2}}\right] \right) \operatorname{Log}\left[1 + \frac{(-b + \sqrt{a^2 + b^2}) e^{c+d x}}{a}\right] - \\
& \frac{1}{2} \left(-2 i c + \pi - 2 i d x - 4 \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i b}{a}}}{\sqrt{2}}\right] \right) \operatorname{Log}\left[1 - \frac{(b + \sqrt{a^2 + b^2}) e^{c+d x}}{a}\right] + \\
& \left. \left(\frac{\pi}{2} - i (c + d x) \right) \operatorname{Log}[b + a \operatorname{Sinh}[c + d x]] + i \left(\operatorname{PolyLog}\left[2, \frac{(b - \sqrt{a^2 + b^2}) e^{c+d x}}{a}\right] + \right. \right. \\
& \left. \left. \operatorname{PolyLog}\left[2, \frac{(b + \sqrt{a^2 + b^2}) e^{c+d x}}{a}\right] \right) + a d x \operatorname{Sinh}[c + d x] \right)
\end{aligned}$$

Problem 19: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(e + f x) \operatorname{Cosh}[c + d x]}{a + b \operatorname{Csch}[c + d x]} dx$$

Optimal (type 4, 212 leaves, 11 steps):

$$\begin{aligned} & \frac{b (e+f x)^2}{2 a^2 f} - \frac{f \operatorname{Cosh}[c+d x]}{a d^2} - \frac{b (e+f x) \operatorname{Log}\left[1 + \frac{a e^{c+d x}}{b - \sqrt{a^2+b^2}}\right]}{a^2 d} - \frac{b (e+f x) \operatorname{Log}\left[1 + \frac{a e^{c+d x}}{b + \sqrt{a^2+b^2}}\right]}{a^2 d} - \\ & \frac{b f \operatorname{PolyLog}[2, -\frac{a e^{c+d x}}{b - \sqrt{a^2+b^2}}]}{a^2 d^2} - \frac{b f \operatorname{PolyLog}[2, -\frac{a e^{c+d x}}{b + \sqrt{a^2+b^2}}]}{a^2 d^2} + \frac{(e+f x) \operatorname{Sinh}[c+d x]}{a d} \end{aligned}$$

Result (type 4, 401 leaves):

$$\begin{aligned} & -\frac{1}{a^2 d^2 (a + b \operatorname{Csch}[c + d x])} \operatorname{Csch}[c + d x] (b + a \operatorname{Sinh}[c + d x]) \\ & \left(d e (b \operatorname{Log}[b + a \operatorname{Sinh}[c + d x]] - a \operatorname{Sinh}[c + d x]) + \frac{1}{8} f \left(-b (2 c + \frac{i}{2} \pi + 2 d x)^2 - \right. \right. \\ & \left. \left. 32 b \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i b}{a}}}{\sqrt{2}}\right] \operatorname{ArcTan}\left[\frac{(\frac{i}{2} a + b) \operatorname{Cot}\left[\frac{1}{4} (2 \frac{i}{2} c + \pi + 2 i d x)\right]}{\sqrt{a^2 + b^2}}\right] + 8 a \operatorname{Cosh}[c + d x] + \right. \right. \\ & \left. \left. 4 b \left(2 c + \frac{i}{2} \pi + 2 d x + 4 \frac{i}{2} \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i b}{a}}}{\sqrt{2}}\right] \right) \operatorname{Log}\left[1 + \frac{(-b + \sqrt{a^2 + b^2}) e^{c+d x}}{a}\right] + \right. \right. \\ & \left. \left. 4 b \left(2 c + \frac{i}{2} \pi + 2 d x - 4 \frac{i}{2} \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i b}{a}}}{\sqrt{2}}\right] \right) \operatorname{Log}\left[1 - \frac{(b + \sqrt{a^2 + b^2}) e^{c+d x}}{a}\right] - \right. \right. \\ & \left. \left. 4 \frac{i}{2} b \pi \operatorname{Log}[b + a \operatorname{Sinh}[c + d x]] - 8 b c \operatorname{Log}\left[1 + \frac{a \operatorname{Sinh}[c + d x]}{b}\right] + \right. \right. \\ & \left. \left. 8 b \left(\operatorname{PolyLog}[2, \frac{(b - \sqrt{a^2 + b^2}) e^{c+d x}}{a}] + \operatorname{PolyLog}[2, \frac{(b + \sqrt{a^2 + b^2}) e^{c+d x}}{a}] \right) - \right. \right. \\ & \left. \left. 8 a d x \operatorname{Sinh}[c + d x] \right) \right) \end{aligned}$$

Problem 21: Attempted integration timed out after 120 seconds.

$$\int \frac{\operatorname{Cosh}[c+d x]}{(e+f x) (a+b \operatorname{Csch}[c+d x])} dx$$

Optimal (type 8, 35 leaves, 1 step):

$$\operatorname{Int}\left[\frac{\operatorname{Cosh}[c+d x] \operatorname{Sinh}[c+d x]}{(e+f x) (b+a \operatorname{Sinh}[c+d x])}, x\right]$$

Result (type 1, 1 leaves):

???

Problem 22: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(e+f x)^3 \operatorname{Cosh}[c+d x]^2}{a+b \operatorname{Csch}[c+d x]} dx$$

Optimal (type 4, 696 leaves, 24 steps):

$$\begin{aligned} & \frac{3 e f^2 x}{4 a d^2} + \frac{3 f^3 x^2}{8 a d^2} + \frac{(e+f x)^4}{8 a f} + \frac{b^2 (e+f x)^4}{4 a^3 f} - \\ & \frac{6 b f^2 (e+f x) \operatorname{Cosh}[c+d x]}{a^2 d^3} - \frac{b (e+f x)^3 \operatorname{Cosh}[c+d x]}{a^2 d} - \frac{3 f^3 \operatorname{Cosh}[c+d x]^2}{8 a d^4} - \\ & \frac{3 f (e+f x)^2 \operatorname{Cosh}[c+d x]^2}{4 a d^2} - \frac{b \sqrt{a^2+b^2} (e+f x)^3 \operatorname{Log}\left[1+\frac{a e^{c+d x}}{b-\sqrt{a^2+b^2}}\right]}{a^3 d} + \\ & \frac{b \sqrt{a^2+b^2} (e+f x)^3 \operatorname{Log}\left[1+\frac{a e^{c+d x}}{b+\sqrt{a^2+b^2}}\right]}{a^3 d} - \frac{3 b \sqrt{a^2+b^2} f (e+f x)^2 \operatorname{PolyLog}[2, -\frac{a e^{c+d x}}{b-\sqrt{a^2+b^2}}]}{a^3 d^2} + \\ & \frac{3 b \sqrt{a^2+b^2} f (e+f x)^2 \operatorname{PolyLog}[2, -\frac{a e^{c+d x}}{b+\sqrt{a^2+b^2}}]}{a^3 d^2} + \frac{6 b \sqrt{a^2+b^2} f^2 (e+f x) \operatorname{PolyLog}[3, -\frac{a e^{c+d x}}{b-\sqrt{a^2+b^2}}]}{a^3 d^3} - \\ & \frac{6 b \sqrt{a^2+b^2} f^2 (e+f x) \operatorname{PolyLog}[3, -\frac{a e^{c+d x}}{b+\sqrt{a^2+b^2}}]}{a^3 d^3} - \frac{6 b \sqrt{a^2+b^2} f^3 \operatorname{PolyLog}[4, -\frac{a e^{c+d x}}{b-\sqrt{a^2+b^2}}]}{a^3 d^4} + \\ & \frac{6 b \sqrt{a^2+b^2} f^3 \operatorname{PolyLog}[4, -\frac{a e^{c+d x}}{b+\sqrt{a^2+b^2}}]}{a^3 d^4} + \frac{6 b f^3 \operatorname{Sinh}[c+d x]}{a^2 d^4} + \frac{3 b f (e+f x)^2 \operatorname{Sinh}[c+d x]}{a^2 d^2} + \\ & \frac{3 f^2 (e+f x) \operatorname{Cosh}[c+d x] \operatorname{Sinh}[c+d x]}{4 a d^3} + \frac{(e+f x)^3 \operatorname{Cosh}[c+d x] \operatorname{Sinh}[c+d x]}{2 a d} \end{aligned}$$

Result (type 4, 3560 leaves):

$$\begin{aligned} & \frac{e^3 \left(\frac{c}{d} + x - \frac{2 b \operatorname{ArcTan}\left[\frac{a-b \operatorname{Tanh}\left[\frac{1}{2} (c+d x)\right]}{\sqrt{-a^2-b^2}}\right]}{\sqrt{-a^2-b^2} d} \right) \operatorname{Csch}[c+d x] (b+a \operatorname{Sinh}[c+d x])}{4 a (a+b \operatorname{Csch}[c+d x])} + \end{aligned}$$

$$\begin{aligned}
& \frac{1}{8 a (a + b \operatorname{Csch}[c + d x])} 3 e^2 f \operatorname{Csch}[c + d x] \left(x^2 + \frac{1}{d^2} 2 b \left(\frac{\frac{i \pi \operatorname{ArcTanh}\left[\frac{-a+b \operatorname{Tanh}\left[\frac{1}{2} (c+d x)\right]}{\sqrt{a^2+b^2}}\right]}{\sqrt{a^2+b^2}} + \right. \right. \\
& \left. \left. \frac{1}{\sqrt{-a^2-b^2}} \left(2 \left(c + i \operatorname{ArcCos}\left[-\frac{i b}{a}\right] \right) \operatorname{ArcTan}\left[\frac{(a-i b) \operatorname{Cot}\left[\frac{1}{4} (2 i c + \pi + 2 i d x)\right]}{\sqrt{-a^2-b^2}}\right] + \right. \right. \\
& \left. \left. (-2 i c + \pi - 2 i d x) \operatorname{ArcTanh}\left[\frac{(-i a + b) \operatorname{Tan}\left[\frac{1}{4} (2 i c + \pi + 2 i d x)\right]}{\sqrt{-a^2-b^2}}\right] - \right. \right. \\
& \left. \left. \left(\operatorname{ArcCos}\left[-\frac{i b}{a}\right] - 2 \operatorname{ArcTan}\left[\frac{(a-i b) \operatorname{Cot}\left[\frac{1}{4} (2 i c + \pi + 2 i d x)\right]}{\sqrt{-a^2-b^2}}\right] \right) \right. \right. \\
& \left. \left. \operatorname{Log}\left[\left((a+i b) \left(a-i b + \sqrt{-a^2-b^2}\right) \left(1 + i \operatorname{Cot}\left[\frac{1}{4} (2 i c + \pi + 2 i d x)\right]\right) \right) \right] / \right. \right. \\
& \left. \left. \left(a \left(a+i b + i \sqrt{-a^2-b^2} \operatorname{Cot}\left[\frac{1}{4} (2 i c + \pi + 2 i d x)\right]\right) \right) \right] - \right. \right. \\
& \left. \left. \left(\operatorname{ArcCos}\left[-\frac{i b}{a}\right] + 2 \operatorname{ArcTan}\left[\frac{(a-i b) \operatorname{Cot}\left[\frac{1}{4} (2 i c + \pi + 2 i d x)\right]}{\sqrt{-a^2-b^2}}\right] \right) \right. \right. \\
& \left. \left. \operatorname{Log}\left[\left(i (a+i b) \left(-a+i b + \sqrt{-a^2-b^2}\right) \left(i + \operatorname{Cot}\left[\frac{1}{4} (2 i c + \pi + 2 i d x)\right]\right) \right) \right] / \right. \right. \\
& \left. \left. \left(a \left(a+i b + i \sqrt{-a^2-b^2} \operatorname{Cot}\left[\frac{1}{4} (2 i c + \pi + 2 i d x)\right]\right) \right) \right] + \right. \right. \\
& \left. \left. \left(\operatorname{ArcCos}\left[-\frac{i b}{a}\right] + 2 \operatorname{ArcTan}\left[\frac{(a-i b) \operatorname{Cot}\left[\frac{1}{4} (2 i c + \pi + 2 i d x)\right]}{\sqrt{-a^2-b^2}}\right] - 2 \right. \right. \\
& \left. \left. i \operatorname{ArcTanh}\left[\frac{(-i a + b) \operatorname{Tan}\left[\frac{1}{4} (2 i c + \pi + 2 i d x)\right]}{\sqrt{-a^2-b^2}}\right] \right) \right. \right. \\
& \left. \left. \operatorname{Log}\left[\frac{\sqrt{-a^2-b^2} e^{\frac{1}{4} (-2 c-i \pi-2 d x)}}{\sqrt{2} \sqrt{-i a} \sqrt{b+a \operatorname{Sinh}[c+d x]}} \right] + \right. \right. \\
& \left. \left. \left(\operatorname{ArcCos}\left[-\frac{i b}{a}\right] - 2 \operatorname{ArcTan}\left[\frac{(a-i b) \operatorname{Cot}\left[\frac{1}{4} (2 i c + \pi + 2 i d x)\right]}{\sqrt{-a^2-b^2}}\right] + 2 \right. \right. \\
& \left. \left. i \operatorname{ArcTanh}\left[\frac{(-i a + b) \operatorname{Tan}\left[\frac{1}{4} (2 i c + \pi + 2 i d x)\right]}{\sqrt{-a^2-b^2}}\right] \right) \right. \right. \\
& \left. \left. \operatorname{Log}\left[\frac{\sqrt{-a^2-b^2} e^{\frac{1}{4} (2 c+i \pi+2 d x)}}{\sqrt{2} \sqrt{-i a} \sqrt{b+a \operatorname{Sinh}[c+d x]}} \right] + i \left(\operatorname{PolyLog}[2, \right. \right. \\
& \left. \left. \left(i b + \sqrt{-a^2-b^2} \right) \left(a+i b - i \sqrt{-a^2-b^2} \operatorname{Cot}\left[\frac{1}{4} (2 i c + \pi + 2 i d x)\right]\right) \right) \right] / \right. \right. \\
& \left. \left. \left(a \left(a+i b + i \sqrt{-a^2-b^2} \operatorname{Cot}\left[\frac{1}{4} (2 i c + \pi + 2 i d x)\right]\right) \right) \right] - \operatorname{PolyLog}[2,
\right. \right. \\
\end{aligned}$$

$$\begin{aligned}
& \left(\left(b + i \sqrt{-a^2 - b^2} \right) \left(i a - b + \sqrt{-a^2 - b^2} \cot \left[\frac{1}{4} (2 i c + \pi + 2 i d x) \right] \right) \right) / \\
& \left(a \left(a + i b + i \sqrt{-a^2 - b^2} \cot \left[\frac{1}{4} (2 i c + \pi + 2 i d x) \right] \right) \right) \Bigg) \\
& (b + a \sinh[c + d x]) + \left(e f^2 \operatorname{Csch}[c + d x] \left(x^3 - \left(3 b e^c \left(d^2 x^2 \operatorname{Log} \left[1 + \frac{a e^{2 c+d x}}{b e^c - \sqrt{(a^2 + b^2) e^{2 c}}} \right] - \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. d^2 x^2 \operatorname{Log} \left[1 + \frac{a e^{2 c+d x}}{b e^c + \sqrt{(a^2 + b^2) e^{2 c}}} \right] + 2 d x \operatorname{PolyLog} \left[2, - \frac{a e^{2 c+d x}}{b e^c - \sqrt{(a^2 + b^2) e^{2 c}}} \right] - \right. \right. \right. \right. \\
& \left. \left. \left. \left. 2 d x \operatorname{PolyLog} \left[2, - \frac{a e^{2 c+d x}}{b e^c + \sqrt{(a^2 + b^2) e^{2 c}}} \right] - 2 \operatorname{PolyLog} \left[3, - \frac{a e^{2 c+d x}}{b e^c - \sqrt{(a^2 + b^2) e^{2 c}}} \right] + \right. \right. \right. \right. \\
& \left. \left. \left. \left. 2 \operatorname{PolyLog} \left[3, - \frac{a e^{2 c+d x}}{b e^c + \sqrt{(a^2 + b^2) e^{2 c}}} \right] \right) \right) \Bigg) / \left(d^3 \sqrt{(a^2 + b^2) e^{2 c}} \right) \right) \\
& (b + a \sinh[c + d x]) \Bigg) / (4 a (a + b \operatorname{Csch}[c + d x])) + \left(f^3 \right. \\
& \operatorname{Csch}[\\
& c + \\
& d \\
& x] \\
& \left(x^4 - \left(4 b e^c \left(d^3 x^3 \operatorname{Log} \left[1 + \frac{a e^{2 c+d x}}{b e^c - \sqrt{(a^2 + b^2) e^{2 c}}} \right] - d^3 x^3 \operatorname{Log} \left[1 + \frac{a e^{2 c+d x}}{b e^c + \sqrt{(a^2 + b^2) e^{2 c}}} \right] + 3 d^2 \right. \right. \right. \right. \\
& \left. \left. \left. x^2 \operatorname{PolyLog} \left[2, - \frac{a e^{2 c+d x}}{b e^c - \sqrt{(a^2 + b^2) e^{2 c}}} \right] - 3 d^2 x^2 \operatorname{PolyLog} \left[2, - \frac{a e^{2 c+d x}}{b e^c + \sqrt{(a^2 + b^2) e^{2 c}}} \right] - \right. \right. \right. \\
& \left. \left. \left. 6 d x \operatorname{PolyLog} \left[3, - \frac{a e^{2 c+d x}}{b e^c - \sqrt{(a^2 + b^2) e^{2 c}}} \right] + 6 d x \operatorname{PolyLog} \left[3, - \frac{a e^{2 c+d x}}{b e^c + \sqrt{(a^2 + b^2) e^{2 c}}} \right] + \right. \right. \right. \\
& \left. \left. \left. 6 \operatorname{PolyLog} \left[4, - \frac{a e^{2 c+d x}}{b e^c - \sqrt{(a^2 + b^2) e^{2 c}}} \right] - 6 \operatorname{PolyLog} \left[4, - \frac{a e^{2 c+d x}}{b e^c + \sqrt{(a^2 + b^2) e^{2 c}}} \right] \right) \right) \Bigg) / \\
& \left(d^4 \sqrt{(a^2 + b^2) e^{2 c}} \right) \Bigg) (b + a \sinh[c + d x]) \Bigg) / \\
& (16 a (a + b \operatorname{Csch}[c + d x])) + \frac{1}{8 a^3 (a + b \operatorname{Csch}[c + d x])} \\
& e \\
& f^2 \\
& \operatorname{Csch}[c + d x]
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{c}{d} x \right] \\
& \left(2 (a^2 + 4 b^2) x^3 - \left(6 b (3 a^2 + 4 b^2) e^c \left(d^2 x^2 \operatorname{Log} \left[1 + \frac{a e^{2 c+d x}}{b e^c - \sqrt{(a^2 + b^2) e^{2 c}}} \right] - \right. \right. \right. \\
& \quad \left. \left. \left. d^2 x^2 \operatorname{Log} \left[1 + \frac{a e^{2 c+d x}}{b e^c + \sqrt{(a^2 + b^2) e^{2 c}}} \right] + 2 d x \operatorname{PolyLog} [2, -\frac{a e^{2 c+d x}}{b e^c - \sqrt{(a^2 + b^2) e^{2 c}}}] - \right. \right. \right. \\
& \quad \left. \left. \left. 2 d x \operatorname{PolyLog} [2, -\frac{a e^{2 c+d x}}{b e^c + \sqrt{(a^2 + b^2) e^{2 c}}}] - \right. \right. \right. \\
& \quad \left. \left. \left. 2 \operatorname{PolyLog} [3, -\frac{a e^{2 c+d x}}{b e^c - \sqrt{(a^2 + b^2) e^{2 c}}}] + 2 \operatorname{PolyLog} [3, -\frac{a e^{2 c+d x}}{b e^c + \sqrt{(a^2 + b^2) e^{2 c}}}] \right) \right) \right) / \\
& \quad \left(d^3 \sqrt{(a^2 + b^2) e^{2 c}} \right) - \frac{24 a b \operatorname{Cosh} [d x] ((2 + d^2 x^2) \operatorname{Cosh} [c] - 2 d x \operatorname{Sinh} [c])}{d^3} + \\
& \quad \frac{3 a^2 \operatorname{Cosh} [2 d x] (-2 d x \operatorname{Cosh} [2 c] + (1 + 2 d^2 x^2) \operatorname{Sinh} [2 c])}{d^3} - \\
& \quad \frac{24 a b (-2 d x \operatorname{Cosh} [c] + (2 + d^2 x^2) \operatorname{Sinh} [c]) \operatorname{Sinh} [d x]}{d^3} + \\
& \quad \left. \frac{3 a^2 ((1 + 2 d^2 x^2) \operatorname{Cosh} [2 c] - 2 d x \operatorname{Sinh} [2 c]) \operatorname{Sinh} [2 d x]}{d^3} \right) \\
& \quad (b + a \operatorname{Sinh} [c + d x]) + \frac{1}{16 a^3 (a + b \operatorname{Csch} [c + d x])} \\
f^3 \\
& \operatorname{Csch} [\\
& \quad c + \\
& \quad d x] \\
& \left((a^2 + 4 b^2) x^4 - \frac{1}{d^4 \sqrt{(a^2 + b^2) e^{2 c}}} 4 b (3 a^2 + 4 b^2) e^c \right. \\
& \quad \left(d^3 x^3 \operatorname{Log} \left[1 + \frac{a e^{2 c+d x}}{b e^c - \sqrt{(a^2 + b^2) e^{2 c}}} \right] - d^3 x^3 \operatorname{Log} \left[1 + \frac{a e^{2 c+d x}}{b e^c + \sqrt{(a^2 + b^2) e^{2 c}}} \right] + \right. \\
& \quad \left. 3 d^2 x^2 \operatorname{PolyLog} [2, -\frac{a e^{2 c+d x}}{b e^c - \sqrt{(a^2 + b^2) e^{2 c}}}] - 3 d^2 x^2 \operatorname{PolyLog} [2, -\frac{a e^{2 c+d x}}{b e^c + \sqrt{(a^2 + b^2) e^{2 c}}}] - \right. \\
& \quad \left. 6 d x \operatorname{PolyLog} [3, -\frac{a e^{2 c+d x}}{b e^c - \sqrt{(a^2 + b^2) e^{2 c}}}] + 6 d x \operatorname{PolyLog} [3, -\frac{a e^{2 c+d x}}{b e^c + \sqrt{(a^2 + b^2) e^{2 c}}}] + \right. \\
& \quad \left. 6 \operatorname{PolyLog} [4, -\frac{a e^{2 c+d x}}{b e^c - \sqrt{(a^2 + b^2) e^{2 c}}}] - 6 \operatorname{PolyLog} [4, -\frac{a e^{2 c+d x}}{b e^c + \sqrt{(a^2 + b^2) e^{2 c}}}] \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{16 a b \cosh[d x] \left(d x \left(6 + d^2 x^2\right) \cosh[c] - 3 \left(2 + d^2 x^2\right) \sinh[c]\right)}{d^4} + \\
& \frac{1}{d^4} \\
& a^2 \cosh[2 d x] \\
& \quad \left(-3 \left(1 + 2 d^2 x^2\right) \cosh[2 c] + 2 d x \left(3 + 2 d^2 x^2\right) \sinh[2 c]\right) - \\
& \frac{1}{d^4} 16 a b \left(-3 \left(2 + d^2 x^2\right) \cosh[c] + d x \left(6 + d^2 x^2\right) \sinh[c]\right) \sinh[d x] + \\
& \frac{1}{d^4} \\
& a^2 \left(2 d x \left(3 + 2 d^2 x^2\right) \cosh[2 c] - 3 \left(1 + 2 d^2 x^2\right) \sinh[2 c]\right) \sinh[2 d x] \Bigg) \\
& \left(b + a \sinh[c + d x]\right) + e^3 \\
& \csc[c + d x] \left(b + a \sinh[c + d x]\right) \\
& \left(\left(a^2 + 4 b^2\right) \left(c + d x\right) - \frac{2 b \left(3 a^2 + 4 b^2\right) \operatorname{ArcTan}\left[\frac{a-b \tanh\left[\frac{1}{2} (c+d x)\right]}{\sqrt{-a^2-b^2}}\right]}{\sqrt{-a^2-b^2}} - \right. \\
& \left. 4 a b \cosh[c + d x] + a^2 \sinh[2 (c + d x)]\right) \Bigg) \Bigg) / \\
& \left(4 a^3 d \left(a + b \csc[c + d x]\right)\right) + \\
& 3 \\
& e^2 \\
& f \\
& \csc[c + d x] \\
& \left(b + a \sinh[c + d x]\right) \\
& \left(\left(a^2 + 4 b^2\right) \left(-c + d x\right) \left(c + d x\right) - \right. \\
& \left. 8 a b d x \cosh[c + d x] - a^2 \cosh[2 (c + d x)] - \right.
\end{aligned}$$

$$\begin{aligned}
& 4 b (3 a^2 + 4 b^2) \left(-\frac{c \operatorname{ArcTan} \left[\frac{b+a e^{c+d x}}{\sqrt{-a^2-b^2}} \right]}{\sqrt{-a^2-b^2}} + \frac{1}{2 \sqrt{a^2+b^2}} \right. \\
& \left((c+d x) \left(\operatorname{Log} \left[1 + \frac{a e^{c+d x}}{b-\sqrt{a^2+b^2}} \right] - \operatorname{Log} \left[1 + \frac{a e^{c+d x}}{b+\sqrt{a^2+b^2}} \right] \right) + \right. \\
& \left. \operatorname{PolyLog} \left[2, \frac{a e^{c+d x}}{-b+\sqrt{a^2+b^2}} \right] - \operatorname{PolyLog} \left[2, \frac{a e^{c+d x}}{b+\sqrt{a^2+b^2}} \right] \right) + \\
& \left. \left. \frac{8 a b \operatorname{Sinh} [c+d x] + 2 a^2 d x \operatorname{Sinh} [2 (c+d x)]}{(8 a^3 d^2 (a+b \operatorname{Csch} [c+d x]))} \right) \right)
\end{aligned}$$

Problem 23: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(e+f x)^2 \operatorname{Cosh} [c+d x]^2}{a+b \operatorname{Csch} [c+d x]} dx$$

Optimal (type 4, 510 leaves, 21 steps):

$$\begin{aligned}
& \frac{f^2 x}{4 a d^2} + \frac{(e+f x)^3}{6 a f} + \frac{b^2 (e+f x)^3}{3 a^3 f} - \frac{2 b f^2 \operatorname{Cosh} [c+d x]}{a^2 d^3} - \frac{b (e+f x)^2 \operatorname{Cosh} [c+d x]}{a^2 d} - \\
& \frac{f (e+f x) \operatorname{Cosh} [c+d x]^2}{2 a d^2} - \frac{b \sqrt{a^2+b^2} (e+f x)^2 \operatorname{Log} \left[1 + \frac{a e^{c+d x}}{b-\sqrt{a^2+b^2}} \right]}{a^3 d} + \\
& \frac{b \sqrt{a^2+b^2} (e+f x)^2 \operatorname{Log} \left[1 + \frac{a e^{c+d x}}{b+\sqrt{a^2+b^2}} \right]}{a^3 d} - \frac{2 b \sqrt{a^2+b^2} f (e+f x) \operatorname{PolyLog} \left[2, -\frac{a e^{c+d x}}{b-\sqrt{a^2+b^2}} \right]}{a^3 d^2} + \\
& \frac{2 b \sqrt{a^2+b^2} f (e+f x) \operatorname{PolyLog} \left[2, -\frac{a e^{c+d x}}{b+\sqrt{a^2+b^2}} \right]}{a^3 d^2} + \frac{2 b \sqrt{a^2+b^2} f^2 \operatorname{PolyLog} \left[3, -\frac{a e^{c+d x}}{b-\sqrt{a^2+b^2}} \right]}{a^3 d^3} - \\
& \frac{2 b \sqrt{a^2+b^2} f^2 \operatorname{PolyLog} \left[3, -\frac{a e^{c+d x}}{b+\sqrt{a^2+b^2}} \right]}{a^3 d^3} + \frac{2 b f (e+f x) \operatorname{Sinh} [c+d x]}{a^2 d^2} + \\
& \frac{f^2 \operatorname{Cosh} [c+d x] \operatorname{Sinh} [c+d x]}{4 a d^3} + \frac{(e+f x)^2 \operatorname{Cosh} [c+d x] \operatorname{Sinh} [c+d x]}{2 a d}
\end{aligned}$$

Result (type 4, 2497 leaves):

$$\begin{aligned}
& \frac{e^2 \left(\frac{c}{d} + x - \frac{2 b \operatorname{ArcTan} \left[\frac{a-b \operatorname{Tanh} \left[\frac{1}{2} (c+d x) \right]}{\sqrt{-a^2-b^2}} \right]}{\sqrt{-a^2-b^2} d} \right) \operatorname{Csch} [c+d x] (b+a \operatorname{Sinh} [c+d x])}{4 a (a+b \operatorname{Csch} [c+d x])} + \\
& \frac{1}{4 a (a+b \operatorname{Csch} [c+d x])} e f \operatorname{Csch} [c+d x] \left(x^2 + \frac{1}{d^2} 2 b \left(\frac{\frac{1}{i \pi} \operatorname{ArcTanh} \left[\frac{-a+b \operatorname{Tanh} \left[\frac{1}{2} (c+d x) \right]}{\sqrt{a^2+b^2}} \right]}{\sqrt{a^2+b^2}} + \right. \right. \\
& \left. \left. \frac{1}{\sqrt{-a^2-b^2}} \left(2 \left(c + i \operatorname{ArcCos} \left[-\frac{i b}{a} \right] \right) \operatorname{ArcTan} \left[\frac{(a-i b) \operatorname{Cot} \left[\frac{1}{4} (2 i c + \pi + 2 i d x) \right]}{\sqrt{-a^2-b^2}} \right] + \right. \right. \\
& \left. \left. (-2 i c + \pi - 2 i d x) \operatorname{ArcTanh} \left[\frac{(-i a+b) \operatorname{Tan} \left[\frac{1}{4} (2 i c + \pi + 2 i d x) \right]}{\sqrt{-a^2-b^2}} \right] - \right. \right. \\
& \left. \left. \left(\operatorname{ArcCos} \left[-\frac{i b}{a} \right] - 2 \operatorname{ArcTan} \left[\frac{(a-i b) \operatorname{Cot} \left[\frac{1}{4} (2 i c + \pi + 2 i d x) \right]}{\sqrt{-a^2-b^2}} \right] \right) \right. \right) \\
& \left. \left. \operatorname{Log} \left[\left((a+i b) \left(a - i b + \sqrt{-a^2-b^2} \right) \left(1 + i \operatorname{Cot} \left[\frac{1}{4} (2 i c + \pi + 2 i d x) \right] \right) \right) \right] / \right. \right. \\
& \left. \left. \left(a \left(a + i b + i \sqrt{-a^2-b^2} \operatorname{Cot} \left[\frac{1}{4} (2 i c + \pi + 2 i d x) \right] \right) \right) \right] - \\
& \left. \left. \left(\operatorname{ArcCos} \left[-\frac{i b}{a} \right] + 2 \operatorname{ArcTan} \left[\frac{(a-i b) \operatorname{Cot} \left[\frac{1}{4} (2 i c + \pi + 2 i d x) \right]}{\sqrt{-a^2-b^2}} \right] \right) \right. \right) \\
& \left. \left. \operatorname{Log} \left[\left(i (a+i b) \left(-a + i b + \sqrt{-a^2-b^2} \right) \left(i + \operatorname{Cot} \left[\frac{1}{4} (2 i c + \pi + 2 i d x) \right] \right) \right) \right] / \right. \right. \\
& \left. \left. \left(a \left(a + i b + i \sqrt{-a^2-b^2} \operatorname{Cot} \left[\frac{1}{4} (2 i c + \pi + 2 i d x) \right] \right) \right) \right] + \\
& \left. \left. \left(\operatorname{ArcCos} \left[-\frac{i b}{a} \right] + 2 \operatorname{ArcTan} \left[\frac{(a-i b) \operatorname{Cot} \left[\frac{1}{4} (2 i c + \pi + 2 i d x) \right]}{\sqrt{-a^2-b^2}} \right] - 2 \right. \right. \right. \\
& \left. \left. \left. i \operatorname{ArcTanh} \left[\frac{(-i a+b) \operatorname{Tan} \left[\frac{1}{4} (2 i c + \pi + 2 i d x) \right]}{\sqrt{-a^2-b^2}} \right] \right) \right. \right) \\
& \left. \left. \operatorname{Log} \left[\frac{\sqrt{-a^2-b^2} e^{\frac{1}{4} (-2 c-i \pi-2 d x)}}{\sqrt{2} \sqrt{-i a} \sqrt{b+a \operatorname{Sinh} [c+d x]}} \right] + \right. \right. \\
& \left. \left. \left(\operatorname{ArcCos} \left[-\frac{i b}{a} \right] - 2 \operatorname{ArcTan} \left[\frac{(a-i b) \operatorname{Cot} \left[\frac{1}{4} (2 i c + \pi + 2 i d x) \right]}{\sqrt{-a^2-b^2}} \right] + 2 \right. \right. \right. \\
& \left. \left. \left. i \operatorname{ArcTanh} \left[\frac{(-i a+b) \operatorname{Tan} \left[\frac{1}{4} (2 i c + \pi + 2 i d x) \right]}{\sqrt{-a^2-b^2}} \right] \right) \right. \right)
\end{aligned}$$

$$\begin{aligned}
& \operatorname{Log}\left[\frac{\sqrt{-a^2-b^2} e^{\frac{1}{4}(2 c+\pi+2 d x)}}{\sqrt{2} \sqrt{-i a} \sqrt{b+a \operatorname{Sinh}[c+d x]}}\right]+i\left(\operatorname{PolyLog}[2,\right. \\
& \left.\left(\left(i b+\sqrt{-a^2-b^2}\right)\left(a+i b-i \sqrt{-a^2-b^2} \operatorname{Cot}\left[\frac{1}{4}(2 i c+\pi+2 i d x)\right]\right)\right)\right. \\
& \left.\left.\left(a\left(a+i b+i \sqrt{-a^2-b^2} \operatorname{Cot}\left[\frac{1}{4}(2 i c+\pi+2 i d x)\right]\right)\right]-\operatorname{PolyLog}[2,\right. \\
& \left.\left.\left(b+i \sqrt{-a^2-b^2}\right)\left(i a-b+\sqrt{-a^2-b^2} \operatorname{Cot}\left[\frac{1}{4}(2 i c+\pi+2 i d x)\right]\right)\right)\right] \\
& \left.\left.\left.\left.a\left(a+i b+i \sqrt{-a^2-b^2} \operatorname{Cot}\left[\frac{1}{4}(2 i c+\pi+2 i d x)\right]\right)\right)\right)\right) \\
& (b+a \operatorname{Sinh}[c+d x])+f^2 \operatorname{Csch}[c+d x]\left(x^3-\left(3 b e^c\left(d^2 x^2 \operatorname{Log}\left[1+\frac{a e^{2 c+d x}}{b e^c-\sqrt{(a^2+b^2) e^{2 c}}}\right]-\right.\right.\right. \\
& \left.\left.d^2 x^2 \operatorname{Log}\left[1+\frac{a e^{2 c+d x}}{b e^c+\sqrt{(a^2+b^2) e^{2 c}}}\right]+2 d x \operatorname{PolyLog}[2,-\frac{a e^{2 c+d x}}{b e^c-\sqrt{(a^2+b^2) e^{2 c}}}\right]-\right. \\
& \left.\left.2 d x \operatorname{PolyLog}[2,-\frac{a e^{2 c+d x}}{b e^c+\sqrt{(a^2+b^2) e^{2 c}}}\right]-2 \operatorname{PolyLog}[3,-\frac{a e^{2 c+d x}}{b e^c-\sqrt{(a^2+b^2) e^{2 c}}}\right]+ \\
& \left.\left.2 \operatorname{PolyLog}[3,-\frac{a e^{2 c+d x}}{b e^c+\sqrt{(a^2+b^2) e^{2 c}}}\right]\right)\right)\right. \\
& (b+a \operatorname{Sinh}[c+d x])\Bigg)\Bigg)\Bigg)\Bigg) \\
& \frac{1}{24 a^3 (a+b \operatorname{Csch}[c+d x])}+ \\
& f^2 \\
& \operatorname{Csch}[\\
& c+ \\
& d \\
& x]\left(2\right. \\
& \left.\left(a^2+4 b^2\right)\right. \\
& x^3-\left. \\
& \left(6 b\left(3 a^2+4 b^2\right) e^c\left(d^2 x^2 \operatorname{Log}\left[1+\frac{a e^{2 c+d x}}{b e^c-\sqrt{(a^2+b^2) e^{2 c}}}\right]-\right.\right. \\
& \left.\left.d^2 x^2 \operatorname{Log}\left[1+\frac{a e^{2 c+d x}}{b e^c+\sqrt{(a^2+b^2) e^{2 c}}}\right]+2 d x \operatorname{PolyLog}[2,-\frac{a e^{2 c+d x}}{b e^c-\sqrt{(a^2+b^2) e^{2 c}}}\right]-\right.\right.
\end{aligned}$$

$$\begin{aligned}
& 2 d x \operatorname{PolyLog}[2, -\frac{a e^{2 c+d x}}{b e^c + \sqrt{(a^2 + b^2) e^{2 c}}}] - \\
& \left. \left. 2 \operatorname{PolyLog}[3, -\frac{a e^{2 c+d x}}{b e^c - \sqrt{(a^2 + b^2) e^{2 c}}}] + 2 \operatorname{PolyLog}[3, -\frac{a e^{2 c+d x}}{b e^c + \sqrt{(a^2 + b^2) e^{2 c}}}] \right) \right\} / \\
& \left(d^3 \sqrt{(a^2 + b^2) e^{2 c}} \right) - \frac{24 a b \operatorname{Cosh}[d x] \left((2 + d^2 x^2) \operatorname{Cosh}[c] - 2 d x \operatorname{Sinh}[c] \right)}{d^3} + \\
& \frac{3 a^2 \operatorname{Cosh}[2 d x] \left(-2 d x \operatorname{Cosh}[2 c] + (1 + 2 d^2 x^2) \operatorname{Sinh}[2 c] \right)}{d^3} - \\
& \frac{24 a b \left(-2 d x \operatorname{Cosh}[c] + (2 + d^2 x^2) \operatorname{Sinh}[c] \right) \operatorname{Sinh}[d x]}{d^3} + \\
& \left. \left. \frac{3 a^2 \left((1 + 2 d^2 x^2) \operatorname{Cosh}[2 c] - 2 d x \operatorname{Sinh}[2 c] \right) \operatorname{Sinh}[2 d x]}{d^3} \right) \right. (b + \\
& a \operatorname{Sinh}[c + d x]) + \\
& \left. \left. \left. \left. e^2 \operatorname{Csch}[c + d x] (b + a \operatorname{Sinh}[c + d x]) \right. \right. \right. \right. \\
& \left((a^2 + 4 b^2) (c + d x) - \right. \\
& \left. \left. \left. \left. \frac{2 b \left(3 a^2 + 4 b^2 \right) \operatorname{ArcTan}\left[\frac{a-b \operatorname{Tanh}\left[\frac{1}{2} (c+d x) \right]}{\sqrt{-a^2-b^2}} \right]}{\sqrt{-a^2-b^2}} - \right. \right. \right. \\
& 4 a b \operatorname{Cosh}[c + d x] + \\
& \left. \left. \left. \left. a^2 \operatorname{Sinh}[2 (c + d x)] \right) \right) \right\} / \\
& \left. \left. \left. \left. (4 a^3 d (a + b \operatorname{Csch}[c + d x])) + e^f \operatorname{Csch}[c + d x] (b + a \operatorname{Sinh}[c + d x]) \right) \right. \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left(\left(a^2 + 4 b^2 \right) (-c + d x) (c + d x) - \right. \\
& 8 a b d x \operatorname{Cosh}[c + d x] - a^2 \operatorname{Cosh}[2 (c + d x)] - \\
& 4 b (3 a^2 + 4 b^2) \left(-\frac{c \operatorname{ArcTan}\left[\frac{b+a e^{c+d x}}{\sqrt{-a^2-b^2}}\right]}{\sqrt{-a^2-b^2}} + \frac{1}{2 \sqrt{a^2+b^2}} \right. \\
& \left((c + d x) \left(\operatorname{Log}\left[1 + \frac{a e^{c+d x}}{b - \sqrt{a^2+b^2}}\right] - \operatorname{Log}\left[1 + \frac{a e^{c+d x}}{b + \sqrt{a^2+b^2}}\right] \right) + \right. \\
& \left. \left. \operatorname{PolyLog}\left[2, -\frac{a e^{c+d x}}{b + \sqrt{a^2+b^2}}\right] - \operatorname{PolyLog}\left[2, -\frac{a e^{c+d x}}{b - \sqrt{a^2+b^2}}\right] \right) \right) + \\
& \left. 8 a b \operatorname{Sinh}[c + d x] + 2 a^2 d x \operatorname{Sinh}[2 (c + d x)] \right) \Bigg) / (4 a^3 d^2 (a + b \operatorname{Csch}[c + d x]))
\end{aligned}$$

Problem 24: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(e + f x) \operatorname{Cosh}[c + d x]^2}{a + b \operatorname{Csch}[c + d x]} dx$$

Optimal (type 4, 327 leaves, 16 steps):

$$\begin{aligned}
& \frac{e x}{2 a} + \frac{b^2 e x}{a^3} + \frac{f x^2}{4 a} + \frac{b^2 f x^2}{2 a^3} - \frac{b (e + f x) \operatorname{Cosh}[c + d x]}{a^2 d} - \frac{f \operatorname{Cosh}[c + d x]^2}{4 a d^2} - \\
& \frac{b \sqrt{a^2 + b^2} (e + f x) \operatorname{Log}\left[1 + \frac{a e^{c+d x}}{b - \sqrt{a^2+b^2}}\right]}{a^3 d} + \frac{b \sqrt{a^2 + b^2} (e + f x) \operatorname{Log}\left[1 + \frac{a e^{c+d x}}{b + \sqrt{a^2+b^2}}\right]}{a^3 d} - \\
& \frac{b \sqrt{a^2 + b^2} f \operatorname{PolyLog}\left[2, -\frac{a e^{c+d x}}{b - \sqrt{a^2+b^2}}\right]}{a^3 d^2} + \frac{b \sqrt{a^2 + b^2} f \operatorname{PolyLog}\left[2, -\frac{a e^{c+d x}}{b + \sqrt{a^2+b^2}}\right]}{a^3 d^2} + \\
& \frac{b f \operatorname{Sinh}[c + d x]}{a^2 d^2} + \frac{(e + f x) \operatorname{Cosh}[c + d x] \operatorname{Sinh}[c + d x]}{2 a d}
\end{aligned}$$

Result (type 4, 1663 leaves):

$$\begin{aligned}
& e \left(\frac{c}{d} + x - \frac{2 b \operatorname{ArcTan}\left[\frac{a-b \operatorname{Tanh}\left[\frac{1}{2} (c+d x)\right]}{\sqrt{-a^2-b^2}}\right]}{\sqrt{-a^2-b^2} d} \right) \operatorname{Csch}[c + d x] (b + a \operatorname{Sinh}[c + d x]) \\
& + \frac{4 a (a + b \operatorname{Csch}[c + d x])}{}
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{8 a (a + b \operatorname{Csch}[c + d x])} f \operatorname{Csch}[c + d x] \left(x^2 + \frac{1}{d^2} 2 b \left(\frac{\frac{i \pi \operatorname{ArcTanh}\left[\frac{-a+b \operatorname{Tanh}\left[\frac{1}{2} (c+d x)\right]}{\sqrt{a^2+b^2}}\right]}{\sqrt{a^2+b^2}} + \right. \right. \\
& \left. \left. \frac{1}{\sqrt{-a^2-b^2}} \left(2 \left(c + i \operatorname{ArcCos}\left[-\frac{i b}{a}\right] \right) \operatorname{ArcTan}\left[\frac{(a-i b) \operatorname{Cot}\left[\frac{1}{4} (2 i c + \pi + 2 i d x)\right]}{\sqrt{-a^2-b^2}}\right] + \right. \right. \\
& \left. \left. (-2 i c + \pi - 2 i d x) \operatorname{ArcTanh}\left[\frac{(-i a + b) \operatorname{Tan}\left[\frac{1}{4} (2 i c + \pi + 2 i d x)\right]}{\sqrt{-a^2-b^2}}\right] - \right. \right. \\
& \left. \left. \left(\operatorname{ArcCos}\left[-\frac{i b}{a}\right] - 2 \operatorname{ArcTan}\left[\frac{(a-i b) \operatorname{Cot}\left[\frac{1}{4} (2 i c + \pi + 2 i d x)\right]}{\sqrt{-a^2-b^2}}\right] \right) \right. \right. \\
& \left. \left. \operatorname{Log}\left[\left((a+i b) \left(a-i b + \sqrt{-a^2-b^2}\right) \left(1 + i \operatorname{Cot}\left[\frac{1}{4} (2 i c + \pi + 2 i d x)\right]\right) \right) \right] / \right. \right. \\
& \left. \left. \left(a \left(a+i b + i \sqrt{-a^2-b^2} \operatorname{Cot}\left[\frac{1}{4} (2 i c + \pi + 2 i d x)\right]\right) \right) \right] - \right. \right. \\
& \left. \left. \left(\operatorname{ArcCos}\left[-\frac{i b}{a}\right] + 2 \operatorname{ArcTan}\left[\frac{(a-i b) \operatorname{Cot}\left[\frac{1}{4} (2 i c + \pi + 2 i d x)\right]}{\sqrt{-a^2-b^2}}\right] \right) \right. \right. \\
& \left. \left. \operatorname{Log}\left[\left(i (a+i b) \left(-a+i b + \sqrt{-a^2-b^2}\right) \left(i + \operatorname{Cot}\left[\frac{1}{4} (2 i c + \pi + 2 i d x)\right]\right) \right) \right] / \right. \right. \\
& \left. \left. \left(a \left(a+i b + i \sqrt{-a^2-b^2} \operatorname{Cot}\left[\frac{1}{4} (2 i c + \pi + 2 i d x)\right]\right) \right) \right] + \right. \right. \\
& \left. \left. \left(\operatorname{ArcCos}\left[-\frac{i b}{a}\right] + 2 \operatorname{ArcTan}\left[\frac{(a-i b) \operatorname{Cot}\left[\frac{1}{4} (2 i c + \pi + 2 i d x)\right]}{\sqrt{-a^2-b^2}}\right] - 2 \right. \right. \\
& \left. \left. i \operatorname{ArcTanh}\left[\frac{(-i a + b) \operatorname{Tan}\left[\frac{1}{4} (2 i c + \pi + 2 i d x)\right]}{\sqrt{-a^2-b^2}}\right] \right) \right. \right. \\
& \left. \left. \operatorname{Log}\left[\frac{\sqrt{-a^2-b^2} e^{\frac{1}{4} (-2 c-i \pi-2 d x)}}{\sqrt{2} \sqrt{-i a} \sqrt{b+a} \operatorname{Sinh}[c+d x]} \right] + \right. \right. \\
& \left. \left. \left(\operatorname{ArcCos}\left[-\frac{i b}{a}\right] - 2 \operatorname{ArcTan}\left[\frac{(a-i b) \operatorname{Cot}\left[\frac{1}{4} (2 i c + \pi + 2 i d x)\right]}{\sqrt{-a^2-b^2}}\right] + 2 \right. \right. \\
& \left. \left. i \operatorname{ArcTanh}\left[\frac{(-i a + b) \operatorname{Tan}\left[\frac{1}{4} (2 i c + \pi + 2 i d x)\right]}{\sqrt{-a^2-b^2}}\right] \right) \right. \right. \\
& \left. \left. \operatorname{Log}\left[\frac{\sqrt{-a^2-b^2} e^{\frac{1}{4} (2 c+i \pi+2 d x)}}{\sqrt{2} \sqrt{-i a} \sqrt{b+a} \operatorname{Sinh}[c+d x]} \right] + i \left(\operatorname{PolyLog}[2, \right. \right. \\
& \left. \left. \left(i b + \sqrt{-a^2-b^2} \right) \left(a+i b - i \sqrt{-a^2-b^2} \operatorname{Cot}\left[\frac{1}{4} (2 i c + \pi + 2 i d x)\right]\right) \right) \right] / \right. \right. \\
& \left. \left. \left(a \left(a+i b + i \sqrt{-a^2-b^2} \operatorname{Cot}\left[\frac{1}{4} (2 i c + \pi + 2 i d x)\right]\right) \right) \right] - \operatorname{PolyLog}[2,
\right. \right. \\
\end{aligned}$$

$$\begin{aligned}
& \left(\left(b + \frac{i}{2} \sqrt{-a^2 - b^2} \right) \left(\frac{i}{2} a - b + \sqrt{-a^2 - b^2} \cot \left[\frac{1}{4} (2i c + \pi + 2i d x) \right] \right) \right) / \\
& \left(a \left(a + \frac{i}{2} b + \frac{i}{2} \sqrt{-a^2 - b^2} \cot \left[\frac{1}{4} (2i c + \pi + 2i d x) \right] \right) \right) \Bigg) \\
& \left(b + a \operatorname{Sinh}[c + d x] \right) + \left(\begin{array}{l} e \operatorname{Csch}[c + d x] (b + a \operatorname{Sinh}[c + d x]) \\ (a^2 + 4 b^2) \\ (c + d x) - \\ \frac{2 b (3 a^2 + 4 b^2) \operatorname{ArcTan} \left[\frac{a-b \operatorname{Tanh} \left[\frac{1}{2} (c+d x) \right]}{\sqrt{-a^2 - b^2}} \right]}{\sqrt{-a^2 - b^2}} - \\ \frac{4}{a} \\ \frac{b}{a} \\ \operatorname{Cosh}[c + d x] + a^2 \\ \operatorname{Sinh}[2 (c + d x)] \end{array} \right) \Bigg) / \\
& \left(4 a^3 d (a + b \operatorname{Csch}[c + d x]) \right) + \left(\begin{array}{l} f \operatorname{Csch}[\\ c + \\ d x] (b + \\ a \operatorname{Sinh}[c + d x]) \\ (a^2 + 4 b^2) (-c + d x) (c + d x) - \\ 8 a b d x \operatorname{Cosh}[c + d x] - \\ a^2 \operatorname{Cosh}[2 (c + d x)] - \\ 4 b (3 a^2 + 4 b^2) \end{array} \right)
\end{aligned}$$

$$\begin{aligned}
& \left(-\frac{c \operatorname{ArcTan} \left[\frac{b+a e^{c+d x}}{\sqrt{-a^2-b^2}} \right]}{\sqrt{-a^2-b^2}} + \frac{1}{2 \sqrt{a^2+b^2}} \right. \\
& \left((c+d x) \left(\operatorname{Log} \left[1 + \frac{a e^{c+d x}}{b - \sqrt{a^2+b^2}} \right] - \operatorname{Log} \left[1 + \frac{a e^{c+d x}}{b + \sqrt{a^2+b^2}} \right] \right) + \right. \\
& \left. \operatorname{PolyLog} \left[2, -\frac{a e^{c+d x}}{b + \sqrt{a^2+b^2}} \right] - \operatorname{PolyLog} \left[2, -\frac{a e^{c+d x}}{b - \sqrt{a^2+b^2}} \right] \right) + \\
& \left. \left. \left. 8 a b \operatorname{Sinh} [c+d x] + 2 a^2 d x \operatorname{Sinh} [2 (c+d x)] \right) \right\} \right) / (8 a^3 d^2 (a + \\
& b \operatorname{Csch} [c+d x]))
\end{aligned}$$

Problem 26: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(e+f x)^3 \operatorname{Cosh} [c+d x]^3}{a+b \operatorname{Csch} [c+d x]} dx$$

Optimal (type 4, 864 leaves, 31 steps):

$$\begin{aligned}
& -\frac{3 b f^3 x}{8 a^2 d^3} - \frac{b (e + f x)^3}{4 a^2 d} + \frac{b (a^2 + b^2) (e + f x)^4}{4 a^4 f} - \frac{40 f^3 \operatorname{Cosh}[c + d x]}{9 a d^4} - \\
& \frac{6 b^2 f^3 \operatorname{Cosh}[c + d x]}{a^3 d^4} - \frac{2 f (e + f x)^2 \operatorname{Cosh}[c + d x]}{a d^2} - \frac{3 b^2 f (e + f x)^2 \operatorname{Cosh}[c + d x]}{a^3 d^2} - \\
& \frac{2 f^3 \operatorname{Cosh}[c + d x]^3}{27 a d^4} - \frac{f (e + f x)^2 \operatorname{Cosh}[c + d x]^3}{3 a d^2} - \frac{b (a^2 + b^2) (e + f x)^3 \operatorname{Log}\left[1 + \frac{a e^{c+d x}}{b - \sqrt{a^2 + b^2}}\right]}{a^4 d} - \\
& \frac{b (a^2 + b^2) (e + f x)^3 \operatorname{Log}\left[1 + \frac{a e^{c+d x}}{b + \sqrt{a^2 + b^2}}\right]}{a^4 d} - \frac{3 b (a^2 + b^2) f (e + f x)^2 \operatorname{PolyLog}[2, -\frac{a e^{c+d x}}{b - \sqrt{a^2 + b^2}}]}{a^4 d^2} - \\
& \frac{3 b (a^2 + b^2) f (e + f x)^2 \operatorname{PolyLog}[2, -\frac{a e^{c+d x}}{b + \sqrt{a^2 + b^2}}]}{a^4 d^2} + \frac{6 b (a^2 + b^2) f^2 (e + f x) \operatorname{PolyLog}[3, -\frac{a e^{c+d x}}{b - \sqrt{a^2 + b^2}}]}{a^4 d^3} + \\
& \frac{6 b (a^2 + b^2) f^2 (e + f x) \operatorname{PolyLog}[3, -\frac{a e^{c+d x}}{b + \sqrt{a^2 + b^2}}]}{a^4 d^3} - \frac{6 b (a^2 + b^2) f^3 \operatorname{PolyLog}[4, -\frac{a e^{c+d x}}{b - \sqrt{a^2 + b^2}}]}{a^4 d^4} - \\
& \frac{6 b (a^2 + b^2) f^3 \operatorname{PolyLog}[4, -\frac{a e^{c+d x}}{b + \sqrt{a^2 + b^2}}]}{a^4 d^4} + \frac{40 f^2 (e + f x) \operatorname{Sinh}[c + d x]}{9 a d^3} + \\
& \frac{6 b^2 f^2 (e + f x) \operatorname{Sinh}[c + d x]}{a^3 d^3} + \frac{2 (e + f x)^3 \operatorname{Sinh}[c + d x]}{3 a d} + \frac{b^2 (e + f x)^3 \operatorname{Sinh}[c + d x]}{a^3 d} + \\
& \frac{3 b f^3 \operatorname{Cosh}[c + d x] \operatorname{Sinh}[c + d x]}{8 a^2 d^4} + \frac{3 b f (e + f x)^2 \operatorname{Cosh}[c + d x] \operatorname{Sinh}[c + d x]}{4 a^2 d^2} + \\
& \frac{2 f^2 (e + f x) \operatorname{Cosh}[c + d x]^2 \operatorname{Sinh}[c + d x]}{9 a d^3} + \frac{(e + f x)^3 \operatorname{Cosh}[c + d x]^2 \operatorname{Sinh}[c + d x]}{3 a d} - \\
& \frac{3 b f^2 (e + f x) \operatorname{Sinh}[c + d x]^2}{4 a^2 d^3} - \frac{b (e + f x)^3 \operatorname{Sinh}[c + d x]^2}{2 a^2 d}
\end{aligned}$$

Result (type 4, 5945 leaves):

$$\begin{aligned}
& \frac{1}{4 a^2 d^3 (a + b \operatorname{Csch}[c + d x])} e f^2 \operatorname{Csch}[c + d x] \\
& \left(-12 b d x \operatorname{PolyLog}\left[2, -\frac{a e^{2 c+d x}}{b e^c - \sqrt{(a^2 + b^2) e^{2 c}}}\right] - 12 b d x \operatorname{PolyLog}\left[2, -\frac{a e^{2 c+d x}}{b e^c + \sqrt{(a^2 + b^2) e^{2 c}}}\right] + \right. \\
& \left. e^{-c} \left(2 b d^3 e^c x^3 - 6 a \operatorname{Cosh}[d x] + 6 a e^{2 c} \operatorname{Cosh}[d x] - 6 a d x \operatorname{Cosh}[d x] - 6 a d e^{2 c} x \operatorname{Cosh}[d x] - \right. \right. \\
& \left. \left. 3 a d^2 x^2 \operatorname{Cosh}[d x] + 3 a d^2 e^{2 c} x^2 \operatorname{Cosh}[d x] - 6 b d^2 e^c x^2 \operatorname{Log}\left[1 + \frac{a e^{2 c+d x}}{b e^c - \sqrt{(a^2 + b^2) e^{2 c}}}\right] - \right. \right. \\
& \left. \left. 6 b d^2 e^c x^2 \operatorname{Log}\left[1 + \frac{a e^{2 c+d x}}{b e^c + \sqrt{(a^2 + b^2) e^{2 c}}}\right] + 12 b e^c \operatorname{PolyLog}\left[3, -\frac{a e^{2 c+d x}}{b e^c - \sqrt{(a^2 + b^2) e^{2 c}}}\right] + \right. \right)
\end{aligned}$$

$$\begin{aligned}
& 12 b e^c \operatorname{PolyLog}[3, -\frac{a e^{2 c+d x}}{b e^c + \sqrt{(a^2 + b^2) e^{2 c}}}] + 6 a \operatorname{Sinh}[d x] + 6 a e^{2 c} \operatorname{Sinh}[d x] + \\
& 6 a d x \operatorname{Sinh}[d x] - 6 a d e^{2 c} x \operatorname{Sinh}[d x] + 3 a d^2 x^2 \operatorname{Sinh}[d x] + 3 a d^2 e^{2 c} x^2 \operatorname{Sinh}[d x] \Bigg) \\
& (b + a \operatorname{Sinh}[c + d x]) + \frac{1}{8 a^2 d^4 (a + b \operatorname{Csch}[c + d x])} f^3 \operatorname{Csch}[c + d x] \\
& \left(-12 b d^2 x^2 \operatorname{PolyLog}[2, -\frac{a e^{2 c+d x}}{b e^c - \sqrt{(a^2 + b^2) e^{2 c}}}] + \right. \\
& e^{-c} \left(b d^4 e^c x^4 - 12 a \operatorname{Cosh}[d x] - 12 a e^{2 c} \operatorname{Cosh}[d x] - 12 a d x \operatorname{Cosh}[d x] + 12 a d e^{2 c} x \operatorname{Cosh}[d x] - \right. \\
& 6 a d^2 x^2 \operatorname{Cosh}[d x] - 6 a d^2 e^{2 c} x^2 \operatorname{Cosh}[d x] - 2 a d^3 x^3 \operatorname{Cosh}[d x] + 2 a d^3 e^{2 c} x^3 \operatorname{Cosh}[d x] - \\
& 4 b d^3 e^c x^3 \operatorname{Log}\left[1 + \frac{a e^{2 c+d x}}{b e^c - \sqrt{(a^2 + b^2) e^{2 c}}}\right] - 4 b d^3 e^c x^3 \operatorname{Log}\left[1 + \frac{a e^{2 c+d x}}{b e^c + \sqrt{(a^2 + b^2) e^{2 c}}}\right] - \\
& 12 b d^2 e^c x^2 \operatorname{PolyLog}[2, -\frac{a e^{2 c+d x}}{b e^c + \sqrt{(a^2 + b^2) e^{2 c}}}] + 24 b d e^c x \\
& \operatorname{PolyLog}[3, -\frac{a e^{2 c+d x}}{b e^c - \sqrt{(a^2 + b^2) e^{2 c}}}] + 24 b d e^c x \operatorname{PolyLog}[3, -\frac{a e^{2 c+d x}}{b e^c + \sqrt{(a^2 + b^2) e^{2 c}}}] - \\
& 24 b e^c \operatorname{PolyLog}[4, -\frac{a e^{2 c+d x}}{b e^c - \sqrt{(a^2 + b^2) e^{2 c}}}] - 24 b e^c \operatorname{PolyLog}[4, -\frac{a e^{2 c+d x}}{b e^c + \sqrt{(a^2 + b^2) e^{2 c}}}] + \\
& 12 a \operatorname{Sinh}[d x] - 12 a e^{2 c} \operatorname{Sinh}[d x] + 12 a d x \operatorname{Sinh}[d x] + 12 a d e^{2 c} x \operatorname{Sinh}[d x] + \\
& 6 a d^2 x^2 \operatorname{Sinh}[d x] - 6 a d^2 e^{2 c} x^2 \operatorname{Sinh}[d x] + 2 a d^3 x^3 \operatorname{Sinh}[d x] + 2 a d^3 e^{2 c} x^3 \operatorname{Sinh}[d x] \Bigg) \\
& (b + a \operatorname{Sinh}[c + d x]) + \frac{1}{144 a^4 d^3 (a + b \operatorname{Csch}[c + d x])} e \\
& e^{-3 c} \\
& f^2 \\
& \operatorname{Csch}[c + d x] \\
& \left(72 a^2 b d^3 e^{3 c} x^3 + 144 b^3 d^3 e^{3 c} x^3 - 108 a^3 e^{2 c} \operatorname{Cosh}[d x] - 432 a b^2 e^{2 c} \operatorname{Cosh}[d x] + \right. \\
& 108 a^3 e^{4 c} \operatorname{Cosh}[d x] + 432 a b^2 e^{4 c} \operatorname{Cosh}[d x] - 108 a^3 d e^{2 c} x \operatorname{Cosh}[d x] - \\
& 432 a b^2 d e^{2 c} x \operatorname{Cosh}[d x] - 108 a^3 d e^{4 c} x \operatorname{Cosh}[d x] - 432 a b^2 d e^{4 c} x \operatorname{Cosh}[d x] - \\
& 54 a^3 d^2 e^{2 c} x^2 \operatorname{Cosh}[d x] - 216 a b^2 d^2 e^{2 c} x^2 \operatorname{Cosh}[d x] + 54 a^3 d^2 e^{4 c} x^2 \operatorname{Cosh}[d x] + \\
& 216 a b^2 d^2 e^{4 c} x^2 \operatorname{Cosh}[d x] - 27 a^2 b e^c \operatorname{Cosh}[2 d x] - 27 a^2 b e^{5 c} \operatorname{Cosh}[2 d x] - \\
& 54 a^2 b d e^c x \operatorname{Cosh}[2 d x] + 54 a^2 b d e^{5 c} x \operatorname{Cosh}[2 d x] - 54 a^2 b d^2 e^c x^2 \operatorname{Cosh}[2 d x] - \\
& 54 a^2 b d^2 e^{5 c} x^2 \operatorname{Cosh}[2 d x] - 4 a^3 \operatorname{Cosh}[3 d x] + 4 a^3 e^{6 c} \operatorname{Cosh}[3 d x] - \\
& 12 a^3 d x \operatorname{Cosh}[3 d x] - 12 a^3 d e^{6 c} x \operatorname{Cosh}[3 d x] - 18 a^3 d^2 x^2 \operatorname{Cosh}[3 d x] +
\end{aligned}$$

$$\begin{aligned}
& 18 a^3 d^2 e^{6c} x^2 \operatorname{Cosh}[3 d x] - 216 a^2 b d^2 e^{3c} x^2 \operatorname{Log}\left[1 + \frac{a e^{2c+d x}}{b e^c - \sqrt{(a^2 + b^2) e^{2c}}}\right] - \\
& 432 b^3 d^2 e^{3c} x^2 \operatorname{Log}\left[1 + \frac{a e^{2c+d x}}{b e^c - \sqrt{(a^2 + b^2) e^{2c}}}\right] - \\
& 216 a^2 b d^2 e^{3c} x^2 \operatorname{Log}\left[1 + \frac{a e^{2c+d x}}{b e^c + \sqrt{(a^2 + b^2) e^{2c}}}\right] - \\
& 432 b^3 d^2 e^{3c} x^2 \operatorname{Log}\left[1 + \frac{a e^{2c+d x}}{b e^c + \sqrt{(a^2 + b^2) e^{2c}}}\right] - \\
& 432 b (a^2 + 2 b^2) d e^{3c} x \operatorname{PolyLog}\left[2, -\frac{a e^{2c+d x}}{b e^c - \sqrt{(a^2 + b^2) e^{2c}}}\right] - \\
& 432 b (a^2 + 2 b^2) d e^{3c} x \operatorname{PolyLog}\left[2, -\frac{a e^{2c+d x}}{b e^c + \sqrt{(a^2 + b^2) e^{2c}}}\right] + \\
& 432 a^2 b e^{3c} \operatorname{PolyLog}\left[3, -\frac{a e^{2c+d x}}{b e^c - \sqrt{(a^2 + b^2) e^{2c}}}\right] + \\
& 864 b^3 e^{3c} \operatorname{PolyLog}\left[3, -\frac{a e^{2c+d x}}{b e^c - \sqrt{(a^2 + b^2) e^{2c}}}\right] + \\
& 432 a^2 b e^{3c} \operatorname{PolyLog}\left[3, -\frac{a e^{2c+d x}}{b e^c + \sqrt{(a^2 + b^2) e^{2c}}}\right] + \\
& 864 b^3 e^{3c} \operatorname{PolyLog}\left[3, -\frac{a e^{2c+d x}}{b e^c + \sqrt{(a^2 + b^2) e^{2c}}}\right] + 108 a^3 e^{2c} \operatorname{Sinh}[d x] + \\
& 432 a b^2 e^{2c} \operatorname{Sinh}[d x] + 108 a^3 e^{4c} \operatorname{Sinh}[d x] + 432 a b^2 e^{4c} \operatorname{Sinh}[d x] + \\
& 108 a^3 d e^{2c} x \operatorname{Sinh}[d x] + 432 a b^2 d e^{2c} x \operatorname{Sinh}[d x] - 108 a^3 d e^{4c} x \operatorname{Sinh}[d x] - \\
& 432 a b^2 d e^{4c} x \operatorname{Sinh}[d x] + 54 a^3 d^2 e^{2c} x^2 \operatorname{Sinh}[d x] + 216 a b^2 d^2 e^{2c} x^2 \operatorname{Sinh}[d x] + \\
& 54 a^3 d^2 e^{4c} x^2 \operatorname{Sinh}[d x] + 216 a b^2 d^2 e^{4c} x^2 \operatorname{Sinh}[d x] + 27 a^2 b e^c \operatorname{Sinh}[2 d x] - \\
& 27 a^2 b e^{5c} \operatorname{Sinh}[2 d x] + 54 a^2 b d e^c x \operatorname{Sinh}[2 d x] + 54 a^2 b d e^{5c} x \operatorname{Sinh}[2 d x] + \\
& 54 a^2 b d^2 e^c x^2 \operatorname{Sinh}[2 d x] - 54 a^2 b d^2 e^{5c} x^2 \operatorname{Sinh}[2 d x] + 4 a^3 \operatorname{Sinh}[3 d x] + \\
& 4 a^3 e^{6c} \operatorname{Sinh}[3 d x] + 12 a^3 d x \operatorname{Sinh}[3 d x] - 12 a^3 d e^{6c} x \operatorname{Sinh}[3 d x] + \\
& 18 a^3 d^2 x^2 \operatorname{Sinh}[3 d x] + 18 a^3 d^2 e^{6c} x^2 \operatorname{Sinh}[3 d x] \Bigg) (b + a \operatorname{Sinh}[c + d x]) + \\
& \frac{1}{864 a^4 d^4 (a + b \operatorname{Csch}[c + d x])} e^{-3c} f^3 \operatorname{Csch}[c + d x] \\
& \left(108 a^2 b d^4 e^{3c} x^4 + 216 b^3 d^4 e^{3c} x^4 - 648 a^3 e^{2c} \operatorname{Cosh}[d x] - 2592 a b^2 e^{2c} \operatorname{Cosh}[d x] - \right. \\
& 648 a^3 e^{4c} \operatorname{Cosh}[d x] - 2592 a b^2 e^{4c} \operatorname{Cosh}[d x] - 648 a^3 d e^{2c} x \operatorname{Cosh}[d x] - \\
& 2592 a b^2 d e^{2c} x \operatorname{Cosh}[d x] + 648 a^3 d e^{4c} x \operatorname{Cosh}[d x] + 2592 a b^2 d e^{4c} x \operatorname{Cosh}[d x] - \\
& 324 a^3 d^2 e^{2c} x^2 \operatorname{Cosh}[d x] - 1296 a b^2 d^2 e^{2c} x^2 \operatorname{Cosh}[d x] - 324 a^3 d^2 e^{4c} x^2 \operatorname{Cosh}[d x] - \\
& 1296 a b^2 d^2 e^{4c} x^2 \operatorname{Cosh}[d x] - 108 a^3 d^3 e^{2c} x^3 \operatorname{Cosh}[d x] - 432 a b^2 d^3 e^{2c} x^3 \operatorname{Cosh}[d x] +
\end{aligned}$$

$$\begin{aligned}
& 108 a^3 d^3 e^{4c} x^3 \operatorname{Cosh}[d x] + 432 a b^2 d^3 e^{4c} x^3 \operatorname{Cosh}[d x] - 81 a^2 b e^c \operatorname{Cosh}[2 d x] + \\
& 81 a^2 b e^{5c} \operatorname{Cosh}[2 d x] - 162 a^2 b d e^c x \operatorname{Cosh}[2 d x] - 162 a^2 b d e^{5c} x \operatorname{Cosh}[2 d x] - \\
& 162 a^2 b d^2 e^c x^2 \operatorname{Cosh}[2 d x] + 162 a^2 b d^2 e^{5c} x^2 \operatorname{Cosh}[2 d x] - \\
& 108 a^2 b d^3 e^c x^3 \operatorname{Cosh}[2 d x] - 108 a^2 b d^3 e^{5c} x^3 \operatorname{Cosh}[2 d x] - 8 a^3 \operatorname{Cosh}[3 d x] - \\
& 8 a^3 e^{6c} \operatorname{Cosh}[3 d x] - 24 a^3 d x \operatorname{Cosh}[3 d x] + 24 a^3 d e^{6c} x \operatorname{Cosh}[3 d x] - \\
& 36 a^3 d^2 x^2 \operatorname{Cosh}[3 d x] - 36 a^3 d^2 e^{6c} x^2 \operatorname{Cosh}[3 d x] - 36 a^3 d^3 x^3 \operatorname{Cosh}[3 d x] + \\
& 36 a^3 d^3 e^{6c} x^3 \operatorname{Cosh}[3 d x] - 432 a^2 b d^3 e^{3c} x^3 \operatorname{Log}\left[1 + \frac{a e^{2c+d x}}{b e^c - \sqrt{(a^2 + b^2) e^{2c}}}\right] - \\
& 864 b^3 d^3 e^{3c} x^3 \operatorname{Log}\left[1 + \frac{a e^{2c+d x}}{b e^c - \sqrt{(a^2 + b^2) e^{2c}}}\right] - 432 a^2 b d^3 e^{3c} x^3 \\
& \operatorname{Log}\left[1 + \frac{a e^{2c+d x}}{b e^c + \sqrt{(a^2 + b^2) e^{2c}}}\right] - 864 b^3 d^3 e^{3c} x^3 \operatorname{Log}\left[1 + \frac{a e^{2c+d x}}{b e^c + \sqrt{(a^2 + b^2) e^{2c}}}\right] - \\
& 1296 b (a^2 + 2 b^2) d^2 e^{3c} x^2 \operatorname{PolyLog}[2, -\frac{a e^{2c+d x}}{b e^c - \sqrt{(a^2 + b^2) e^{2c}}}] - \\
& 1296 b (a^2 + 2 b^2) d^2 e^{3c} x^2 \operatorname{PolyLog}[2, -\frac{a e^{2c+d x}}{b e^c + \sqrt{(a^2 + b^2) e^{2c}}}] + \\
& 2592 a^2 b d e^{3c} x \operatorname{PolyLog}[3, -\frac{a e^{2c+d x}}{b e^c - \sqrt{(a^2 + b^2) e^{2c}}}] + \\
& 5184 b^3 d e^{3c} x \operatorname{PolyLog}[3, -\frac{a e^{2c+d x}}{b e^c - \sqrt{(a^2 + b^2) e^{2c}}}] + \\
& 2592 a^2 b d e^{3c} x \operatorname{PolyLog}[3, -\frac{a e^{2c+d x}}{b e^c + \sqrt{(a^2 + b^2) e^{2c}}}] + \\
& 5184 b^3 d e^{3c} x \operatorname{PolyLog}[3, -\frac{a e^{2c+d x}}{b e^c + \sqrt{(a^2 + b^2) e^{2c}}}] - \\
& 2592 a^2 b e^{3c} \operatorname{PolyLog}[4, -\frac{a e^{2c+d x}}{b e^c - \sqrt{(a^2 + b^2) e^{2c}}}] - \\
& 5184 b^3 e^{3c} \operatorname{PolyLog}[4, -\frac{a e^{2c+d x}}{b e^c - \sqrt{(a^2 + b^2) e^{2c}}}] - \\
& 2592 a^2 b e^{3c} \operatorname{PolyLog}[4, -\frac{a e^{2c+d x}}{b e^c + \sqrt{(a^2 + b^2) e^{2c}}}] - \\
& 5184 b^3 e^{3c} \operatorname{PolyLog}[4, -\frac{a e^{2c+d x}}{b e^c + \sqrt{(a^2 + b^2) e^{2c}}}] + 648 a^3 e^{2c} \operatorname{Sinh}[d x] + \\
& 2592 a b^2 e^{2c} \operatorname{Sinh}[d x] - 648 a^3 e^{4c} \operatorname{Sinh}[d x] - 2592 a b^2 e^{4c} \operatorname{Sinh}[d x] + \\
& 648 a^3 d e^{2c} x \operatorname{Sinh}[d x] + 2592 a b^2 d e^{2c} x \operatorname{Sinh}[d x] + 648 a^3 d e^{4c} x \operatorname{Sinh}[d x] + \\
& 2592 a b^2 d e^{4c} x^2 \operatorname{Sinh}[d x] + 324 a^3 d^2 e^{2c} x^2 \operatorname{Sinh}[d x] + 1296 a b^2 d^2 e^{2c} x^2 \operatorname{Sinh}[d x] - \\
& 324 a^3 d^2 e^{4c} x^2 \operatorname{Sinh}[d x] - 1296 a b^2 d^2 e^{4c} x^2 \operatorname{Sinh}[d x] + 108 a^3 d^3 e^{2c} x^3 \operatorname{Sinh}[d x] + \\
& 432 a b^2 d^3 e^{2c} x^3 \operatorname{Sinh}[d x] + 108 a^3 d^3 e^{4c} x^3 \operatorname{Sinh}[d x] + 432 a b^2 d^3 e^{4c} x^3 \operatorname{Sinh}[d x] +
\end{aligned}$$

$$\begin{aligned}
& 81 a^2 b e^c \operatorname{Sinh}[2 d x] + 81 a^2 b e^{5 c} \operatorname{Sinh}[2 d x] + 162 a^2 b d e^c x \operatorname{Sinh}[2 d x] - \\
& 162 a^2 b d e^{5 c} x \operatorname{Sinh}[2 d x] + 162 a^2 b d^2 e^c x^2 \operatorname{Sinh}[2 d x] + 162 a^2 b d^2 e^{5 c} x^2 \operatorname{Sinh}[2 d x] + \\
& 108 a^2 b d^3 e^c x^3 \operatorname{Sinh}[2 d x] - 108 a^2 b d^3 e^{5 c} x^3 \operatorname{Sinh}[2 d x] + \\
& 8 a^3 \operatorname{Sinh}[3 d x] - 8 a^3 e^{6 c} \operatorname{Sinh}[3 d x] + 24 a^3 d x \operatorname{Sinh}[3 d x] + \\
& 24 a^3 d e^{6 c} x \operatorname{Sinh}[3 d x] + 36 a^3 d^2 x^2 \operatorname{Sinh}[3 d x] - 36 a^3 d^2 e^{6 c} x^2 \operatorname{Sinh}[3 d x] + \\
& 36 a^3 d^3 x^3 \operatorname{Sinh}[3 d x] + 36 a^3 d^3 e^{6 c} x^3 \operatorname{Sinh}[3 d x] \Big) \left(b + a \operatorname{Sinh}[c + d x] \right) + \\
& \left(e^3 \operatorname{Csch}[c + d x] \left(b + a \operatorname{Sinh}[c + d x] \right) \left(-\frac{2 b \operatorname{Log}[b + a \operatorname{Sinh}[c + d x]]}{a^2 d} + \frac{2 \operatorname{Sinh}[c + d x]}{a d} \right) \right) / \\
& (4 (a + b \operatorname{Csch}[c + d x])) + \\
& \frac{1}{2 a^2 d^2 (a + b \operatorname{Csch}[c + d x])} \\
& 3 e^2 f \operatorname{Csch}[c + d x] \left(b + a \operatorname{Sinh}[c + d x] \right) \\
& \left(-a \operatorname{Cosh}[c + d x] - b (c + d x) \operatorname{Log}[b + a \operatorname{Sinh}[c + d x]] + \right. \\
& b c \operatorname{Log}\left[1 + \frac{a \operatorname{Sinh}[c + d x]}{b}\right] + \frac{1}{8} b \left(-\frac{1}{8} (2 c + \frac{1}{2} \pi + 2 d x)^2 - \right. \\
& \left. \left. 4 \frac{1}{2} \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{1}{a} b}}{\sqrt{2}}\right] \operatorname{ArcTan}\left[\frac{(\frac{1}{2} a + b) \operatorname{Cot}\left[\frac{1}{4} (2 \frac{1}{2} c + \pi + 2 d x)\right]}{\sqrt{a^2 + b^2}}\right] - \right. \\
& \left. \frac{1}{2} \left(-2 \frac{1}{2} c + \pi - 2 \frac{1}{2} d x + 4 \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{1}{a} b}}{\sqrt{2}}\right]\right) \operatorname{Log}\left[1 + \frac{(-b + \sqrt{a^2 + b^2}) e^{c+d x}}{a}\right] - \right. \\
& \left. \frac{1}{2} \left(-2 \frac{1}{2} c + \pi - 2 \frac{1}{2} d x - 4 \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{1}{a} b}}{\sqrt{2}}\right]\right) \operatorname{Log}\left[1 - \frac{(b + \sqrt{a^2 + b^2}) e^{c+d x}}{a}\right] + \right. \\
& \left. \left(\frac{\pi}{2} - \frac{1}{2} (c + d x) \right) \operatorname{Log}[b + a \operatorname{Sinh}[c + d x]] + \frac{1}{2} \operatorname{PolyLog}\left[2, \frac{(b - \sqrt{a^2 + b^2}) e^{c+d x}}{a}\right] + \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \left(\operatorname{PolyLog}[2, \frac{(b + \sqrt{a^2 + b^2}) e^{c+d x}}{a}] \right) + a d x \operatorname{Sinh}[c + d x] \right) + \\
& \left. \left(e^3 \operatorname{Csch}[c + d x] (b + a \operatorname{Sinh}[c + d x]) \left(-\frac{2 b \operatorname{Cosh}[2(c + d x)]}{a^2 d} - \right. \right. \right. \\
& \left. \left. \left. \frac{4(a^2 b + 2 b^3) \operatorname{Log}[b + a \operatorname{Sinh}[c + d x]]}{a^4 d} + \frac{2(a^2 + 4 b^2) \operatorname{Sinh}[c + d x]}{a^3 d} + \frac{2 \operatorname{Sinh}[3(c + d x)]}{3 a d} \right) \right) \right) / \\
& (8(a + b \operatorname{Csch}[c + d x])) + \frac{1}{24 a^4 d^2 (a + b \operatorname{Csch}[c + d x])} \\
& e^2 f \operatorname{Csch}[c + d x] (b + a \operatorname{Sinh}[c + d x]) \\
& \left. \left(-18 a (a^2 + 4 b^2) \operatorname{Cosh}[c + d x] - 18 a^2 b d x \operatorname{Cosh}[2(c + d x)] - 2 a^3 \operatorname{Cosh}[3(c + d x)] + \right. \right. \\
& 36 a^2 b c \operatorname{Log}\left[1 + \frac{a \operatorname{Sinh}[c + d x]}{b}\right] + 72 b^3 c \operatorname{Log}\left[1 + \frac{a \operatorname{Sinh}[c + d x]}{b}\right] - 36 a^2 b \\
& \left. \left. \left(-\frac{1}{8} (2 c + \frac{i}{2} \pi + 2 d x)^2 - 4 \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i b}{a}}}{\sqrt{2}}\right] \operatorname{ArcTan}\left[\frac{(\frac{i}{2} a + b) \operatorname{Cot}\left[\frac{1}{4}(2 \frac{i}{2} c + \pi + 2 i d x)\right]}{\sqrt{a^2 + b^2}}\right] + \right. \right. \right. \\
& \frac{1}{2} \left(2 c + \frac{i}{2} \pi + 2 d x + 4 \frac{i}{2} \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i b}{a}}}{\sqrt{2}}\right] \right) \operatorname{Log}\left[1 + \frac{(b + \sqrt{a^2 + b^2}) e^{c+d x}}{a}\right] + \\
& \left. \left. \left. \frac{1}{2} \left(2 c + \frac{i}{2} \pi + 2 d x - 4 \frac{i}{2} \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i b}{a}}}{\sqrt{2}}\right] \right) \operatorname{Log}\left[1 - \frac{(b + \sqrt{a^2 + b^2}) e^{c+d x}}{a}\right] - \right. \right. \\
& \left. \left. \left. \frac{1}{2} \frac{i}{2} \pi \operatorname{Log}[b + a \operatorname{Sinh}[c + d x]] + \operatorname{PolyLog}[2, \frac{(b - \sqrt{a^2 + b^2}) e^{c+d x}}{a}] + \right. \right. \right. \\
& \left. \left. \left. \operatorname{PolyLog}[2, \frac{(b + \sqrt{a^2 + b^2}) e^{c+d x}}{a}] \right) - 72 b^3 \right)
\end{aligned}$$

$$\begin{aligned}
& \left(-\frac{1}{8} (2 c + i \pi + 2 d x)^2 - 4 \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i b}{a}}}{\sqrt{2}}\right] \operatorname{ArcTan}\left[\frac{(i a + b) \operatorname{Cot}\left[\frac{1}{4} (2 i c + \pi + 2 i d x)\right]}{\sqrt{a^2 + b^2}}\right] + \right. \\
& \quad \left. \frac{1}{2} \left(2 c + i \pi + 2 d x + 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i b}{a}}}{\sqrt{2}}\right] \right) \operatorname{Log}\left[1 + \frac{(-b + \sqrt{a^2 + b^2}) e^{c+d x}}{a}\right] + \right. \\
& \quad \left. \frac{1}{2} \left(2 c + i \pi + 2 d x - 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i b}{a}}}{\sqrt{2}}\right] \right) \operatorname{Log}\left[1 - \frac{(b + \sqrt{a^2 + b^2}) e^{c+d x}}{a}\right] - \right. \\
& \quad \left. \frac{1}{2} i \pi \operatorname{Log}[b + a \operatorname{Sinh}[c + d x]] + \operatorname{PolyLog}\left[2, \frac{(b - \sqrt{a^2 + b^2}) e^{c+d x}}{a}\right] + \right. \\
& \quad \left. \operatorname{PolyLog}\left[2, \frac{(b + \sqrt{a^2 + b^2}) e^{c+d x}}{a}\right] \right) + \\
& \quad 18 a (a^2 + 4 b^2) d x \operatorname{Sinh}[c + d x] + 9 a^2 b \operatorname{Sinh}[2 (c + d x)] + 6 a^3 d x \operatorname{Sinh}[3 (c + d x)]
\end{aligned}$$

Problem 27: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(e + f x)^2 \operatorname{Cosh}[c + d x]^3}{a + b \operatorname{Csch}[c + d x]} dx$$

Optimal (type 4, 636 leaves, 24 steps):

$$\begin{aligned}
& -\frac{b e f x}{2 a^2 d} - \frac{b f^2 x^2}{4 a^2 d} + \frac{b (a^2 + b^2) (e + f x)^3}{3 a^4 f} - \frac{4 f (e + f x) \operatorname{Cosh}[c + d x]}{3 a d^2} - \frac{2 b^2 f (e + f x) \operatorname{Cosh}[c + d x]}{a^3 d^2} - \\
& \frac{2 f (e + f x) \operatorname{Cosh}[c + d x]^3}{9 a d^2} - \frac{b (a^2 + b^2) (e + f x)^2 \operatorname{Log}\left[1 + \frac{a e^{c+d x}}{b - \sqrt{a^2 + b^2}}\right]}{a^4 d} - \\
& \frac{b (a^2 + b^2) (e + f x)^2 \operatorname{Log}\left[1 + \frac{a e^{c+d x}}{b + \sqrt{a^2 + b^2}}\right]}{a^4 d} - \frac{2 b (a^2 + b^2) f (e + f x) \operatorname{PolyLog}[2, -\frac{a e^{c+d x}}{b - \sqrt{a^2 + b^2}}]}{a^4 d^2} - \\
& \frac{2 b (a^2 + b^2) f (e + f x) \operatorname{PolyLog}[2, -\frac{a e^{c+d x}}{b + \sqrt{a^2 + b^2}}]}{a^4 d^2} + \frac{2 b (a^2 + b^2) f^2 \operatorname{PolyLog}[3, -\frac{a e^{c+d x}}{b - \sqrt{a^2 + b^2}}]}{a^4 d^3} + \\
& \frac{2 b (a^2 + b^2) f^2 \operatorname{PolyLog}[3, -\frac{a e^{c+d x}}{b + \sqrt{a^2 + b^2}}]}{a^4 d^3} + \frac{14 f^2 \operatorname{Sinh}[c + d x]}{9 a d^3} + \\
& \frac{2 b^2 f^2 \operatorname{Sinh}[c + d x]}{a^3 d^3} + \frac{2 (e + f x)^2 \operatorname{Sinh}[c + d x]}{3 a d} + \frac{b^2 (e + f x)^2 \operatorname{Sinh}[c + d x]}{a^3 d} + \\
& \frac{b f (e + f x) \operatorname{Cosh}[c + d x] \operatorname{Sinh}[c + d x]}{2 a^2 d^2} + \frac{(e + f x)^2 \operatorname{Cosh}[c + d x]^2 \operatorname{Sinh}[c + d x]}{3 a d} - \\
& \frac{b f^2 \operatorname{Sinh}[c + d x]^2}{4 a^2 d^3} - \frac{b (e + f x)^2 \operatorname{Sinh}[c + d x]^2}{2 a^2 d} + \frac{2 f^2 \operatorname{Sinh}[c + d x]^3}{27 a d^3}
\end{aligned}$$

Result (type 4, 3303 leaves):

$$\begin{aligned}
& \frac{1}{12 a^2 d^3 (a + b \operatorname{Csch}[c + d x])} f^2 \operatorname{Csch}[c + d x] \\
& \left(-12 b d x \operatorname{PolyLog}\left[2, -\frac{a e^{2 c+d x}}{b e^c - \sqrt{(a^2 + b^2) e^{2 c}}}\right] - 12 b d x \operatorname{PolyLog}\left[2, -\frac{a e^{2 c+d x}}{b e^c + \sqrt{(a^2 + b^2) e^{2 c}}}\right] + \right. \\
& e^{-c} \left(2 b d^3 e^c x^3 - 6 a \operatorname{Cosh}[d x] + 6 a e^{2 c} \operatorname{Cosh}[d x] - 6 a d x \operatorname{Cosh}[d x] - 6 a d e^{2 c} x \operatorname{Cosh}[d x] - \right. \\
& 3 a d^2 x^2 \operatorname{Cosh}[d x] + 3 a d^2 e^{2 c} x^2 \operatorname{Cosh}[d x] - 6 b d^2 e^c x^2 \operatorname{Log}\left[1 + \frac{a e^{2 c+d x}}{b e^c - \sqrt{(a^2 + b^2) e^{2 c}}}\right] - \\
& 6 b d^2 e^c x^2 \operatorname{Log}\left[1 + \frac{a e^{2 c+d x}}{b e^c + \sqrt{(a^2 + b^2) e^{2 c}}}\right] + 12 b e^c \operatorname{PolyLog}\left[3, -\frac{a e^{2 c+d x}}{b e^c - \sqrt{(a^2 + b^2) e^{2 c}}}\right] + \\
& 12 b e^c \operatorname{PolyLog}\left[3, -\frac{a e^{2 c+d x}}{b e^c + \sqrt{(a^2 + b^2) e^{2 c}}}\right] + 6 a \operatorname{Sinh}[d x] + 6 a e^{2 c} \operatorname{Sinh}[d x] + \\
& 6 a d x \operatorname{Sinh}[d x] - 6 a d e^{2 c} x \operatorname{Sinh}[d x] + 3 a d^2 x^2 \operatorname{Sinh}[d x] + 3 a d^2 e^{2 c} x^2 \operatorname{Sinh}[d x] \Big) \\
& \left. \left(b + a \operatorname{Sinh}[c + d x] \right) + \frac{1}{432 a^4 d^3 (a + b \operatorname{Csch}[c + d x])} e^{-3 c} \right)
\end{aligned}$$

$$\begin{aligned}
& f^2 \\
& \operatorname{Csch}[c + d x] \\
& \left(72 a^2 b d^3 e^{3 c} x^3 + 144 b^3 d^3 e^{3 c} x^3 - 108 a^3 e^{2 c} \operatorname{Cosh}[d x] - 432 a b^2 e^{2 c} \operatorname{Cosh}[d x] + \right. \\
& 108 a^3 e^{4 c} \operatorname{Cosh}[d x] + 432 a b^2 e^{4 c} \operatorname{Cosh}[d x] - 108 a^3 d e^{2 c} x \operatorname{Cosh}[d x] - \\
& 432 a b^2 d e^{2 c} x \operatorname{Cosh}[d x] - 108 a^3 d e^{4 c} x \operatorname{Cosh}[d x] - 432 a b^2 d e^{4 c} x \operatorname{Cosh}[d x] - \\
& 54 a^3 d^2 e^{2 c} x^2 \operatorname{Cosh}[d x] - 216 a b^2 d^2 e^{2 c} x^2 \operatorname{Cosh}[d x] + 54 a^3 d^2 e^{4 c} x^2 \operatorname{Cosh}[d x] + \\
& 216 a b^2 d^2 e^{4 c} x^2 \operatorname{Cosh}[d x] - 27 a^2 b e^c \operatorname{Cosh}[2 d x] - 27 a^2 b e^{5 c} \operatorname{Cosh}[2 d x] - \\
& 54 a^2 b d e^c x \operatorname{Cosh}[2 d x] + 54 a^2 b d e^{5 c} x \operatorname{Cosh}[2 d x] - 54 a^2 b d^2 e^c x^2 \operatorname{Cosh}[2 d x] - \\
& 54 a^2 b d^2 e^{5 c} x^2 \operatorname{Cosh}[2 d x] - 4 a^3 \operatorname{Cosh}[3 d x] + 4 a^3 e^{6 c} \operatorname{Cosh}[3 d x] - \\
& 12 a^3 d x \operatorname{Cosh}[3 d x] - 12 a^3 d e^{6 c} x \operatorname{Cosh}[3 d x] - 18 a^3 d^2 x^2 \operatorname{Cosh}[3 d x] + \\
& 18 a^3 d^2 e^{6 c} x^2 \operatorname{Cosh}[3 d x] - 216 a^2 b d^2 e^{3 c} x^2 \operatorname{Log}\left[1 + \frac{a e^{2 c+d x}}{b e^c - \sqrt{(a^2 + b^2) e^{2 c}}}\right] - \\
& 432 b^3 d^2 e^{3 c} x^2 \operatorname{Log}\left[1 + \frac{a e^{2 c+d x}}{b e^c - \sqrt{(a^2 + b^2) e^{2 c}}}\right] - 216 a^2 b d^2 e^{3 c} x^2 \\
& \operatorname{Log}\left[1 + \frac{a e^{2 c+d x}}{b e^c + \sqrt{(a^2 + b^2) e^{2 c}}}\right] - 432 b^3 d^2 e^{3 c} x^2 \operatorname{Log}\left[1 + \frac{a e^{2 c+d x}}{b e^c + \sqrt{(a^2 + b^2) e^{2 c}}}\right] - \\
& 432 b (a^2 + 2 b^2) d e^{3 c} x \operatorname{PolyLog}[2, -\frac{a e^{2 c+d x}}{b e^c - \sqrt{(a^2 + b^2) e^{2 c}}}] - \\
& 432 b (a^2 + 2 b^2) d e^{3 c} x \operatorname{PolyLog}[2, -\frac{a e^{2 c+d x}}{b e^c + \sqrt{(a^2 + b^2) e^{2 c}}}] + \\
& 432 a^2 b e^{3 c} \operatorname{PolyLog}[3, -\frac{a e^{2 c+d x}}{b e^c - \sqrt{(a^2 + b^2) e^{2 c}}}] + \\
& 864 b^3 e^{3 c} \operatorname{PolyLog}[3, -\frac{a e^{2 c+d x}}{b e^c - \sqrt{(a^2 + b^2) e^{2 c}}}] + \\
& 432 a^2 b e^{3 c} \operatorname{PolyLog}[3, -\frac{a e^{2 c+d x}}{b e^c + \sqrt{(a^2 + b^2) e^{2 c}}}] + \\
& 864 b^3 e^{3 c} \operatorname{PolyLog}[3, -\frac{a e^{2 c+d x}}{b e^c + \sqrt{(a^2 + b^2) e^{2 c}}}] + 108 a^3 e^{2 c} \operatorname{Sinh}[d x] + \\
& 432 a b^2 e^{2 c} \operatorname{Sinh}[d x] + 108 a^3 e^{4 c} \operatorname{Sinh}[d x] + 432 a b^2 e^{4 c} \operatorname{Sinh}[d x] + \\
& 108 a^3 d e^{2 c} x \operatorname{Sinh}[d x] + 432 a b^2 d e^{2 c} x \operatorname{Sinh}[d x] - 108 a^3 d e^{4 c} x \operatorname{Sinh}[d x] - \\
& 432 a b^2 d e^{4 c} x \operatorname{Sinh}[d x] + 54 a^3 d^2 e^{2 c} x^2 \operatorname{Sinh}[d x] + 216 a b^2 d^2 e^{2 c} x^2 \operatorname{Sinh}[d x] + \\
& 54 a^3 d^2 e^{4 c} x^2 \operatorname{Sinh}[d x] + 216 a b^2 d^2 e^{4 c} x^2 \operatorname{Sinh}[d x] + 27 a^2 b e^c \operatorname{Sinh}[2 d x] - \\
& 27 a^2 b e^{5 c} \operatorname{Sinh}[2 d x] + 54 a^2 b d e^c x \operatorname{Sinh}[2 d x] + 54 a^2 b d e^{5 c} x \operatorname{Sinh}[2 d x] + \\
& 54 a^2 b d^2 e^c x^2 \operatorname{Sinh}[2 d x] - 54 a^2 b d^2 e^{5 c} x^2 \operatorname{Sinh}[2 d x] + 4 a^3 \operatorname{Sinh}[3 d x] + \\
& 4 a^3 e^{6 c} \operatorname{Sinh}[3 d x] + 12 a^3 d x \operatorname{Sinh}[3 d x] - 12 a^3 d e^{6 c} x \operatorname{Sinh}[3 d x] + \\
& \left. 18 a^3 d^2 x^2 \operatorname{Sinh}[3 d x] + 18 a^3 d^2 e^{6 c} x^2 \operatorname{Sinh}[3 d x] \right) (b + a \operatorname{Sinh}[c + d x]) +
\end{aligned}$$

$$\begin{aligned}
& \left(e^2 \operatorname{Csch}[c+d x] (b+a \operatorname{Sinh}[c+d x]) \left(-\frac{2 b \operatorname{Log}[b+a \operatorname{Sinh}[c+d x]]}{a^2 d} + \frac{2 \operatorname{Sinh}[c+d x]}{a d} \right) \right) / \\
& \frac{(4 (a+b \operatorname{Csch}[c+d x])) + 1}{a^2 d^2 (a+b \operatorname{Csch}[c+d x])} \\
& e f \operatorname{Csch}[c+d x] (b+a \operatorname{Sinh}[c+d x]) \\
& \left(-a \operatorname{Cosh}[c+d x] - b (c+d x) \operatorname{Log}[b+a \operatorname{Sinh}[c+d x]] + \right. \\
& b c \operatorname{Log}\left[1 + \frac{a \operatorname{Sinh}[c+d x]}{b}\right] + \frac{1}{8} b \left(-\frac{1}{8} (2 c + i \pi + 2 d x)^2 - \right. \\
& \left. \left. 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i b}{a}}}{\sqrt{2}}\right] \operatorname{ArcTan}\left[\frac{(i a + b) \operatorname{Cot}\left[\frac{1}{4} (2 i c + \pi + 2 i d x)\right]}{\sqrt{a^2 + b^2}}\right] - \right. \\
& \left. \frac{1}{2} \left(-2 i c + \pi - 2 i d x + 4 \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i b}{a}}}{\sqrt{2}}\right] \right) \operatorname{Log}\left[1 + \frac{(-b + \sqrt{a^2 + b^2}) e^{c+d x}}{a}\right] - \right. \\
& \left. \frac{1}{2} \left(-2 i c + \pi - 2 i d x - 4 \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i b}{a}}}{\sqrt{2}}\right] \right) \operatorname{Log}\left[1 - \frac{(b + \sqrt{a^2 + b^2}) e^{c+d x}}{a}\right] + \right. \\
& \left. \left(\frac{\pi}{2} - i (c+d x) \right) \operatorname{Log}[b+a \operatorname{Sinh}[c+d x]] + i \left(\operatorname{PolyLog}\left[2, \frac{(b - \sqrt{a^2 + b^2}) e^{c+d x}}{a}\right] + \right. \right. \\
& \left. \left. \operatorname{PolyLog}\left[2, \frac{(b + \sqrt{a^2 + b^2}) e^{c+d x}}{a}\right] \right) + a d x \operatorname{Sinh}[c+d x] \right) + \\
& \left(e^2 \operatorname{Csch}[c+d x] (b+a \operatorname{Sinh}[c+d x]) \left(-\frac{2 b \operatorname{Cosh}[2 (c+d x)]}{a^2 d} - \right. \right. \\
& \left. \left. \frac{4 (a^2 b + 2 b^3) \operatorname{Log}[b+a \operatorname{Sinh}[c+d x]]}{a^4 d} + \frac{2 (a^2 + 4 b^2) \operatorname{Sinh}[c+d x]}{a^3 d} + \frac{2 \operatorname{Sinh}[3 (c+d x)]}{3 a d} \right) \right) /
\end{aligned}$$

$$\begin{aligned}
& \left(8 (a + b \operatorname{Csch}[c + d x]) \right) + \frac{1}{36 a^4 d^2 (a + b \operatorname{Csch}[c + d x])} \\
& e f \operatorname{Csch}[c + d x] (b + a \operatorname{Sinh}[c + d x]) \\
& \left(-18 a (a^2 + 4 b^2) \operatorname{Cosh}[c + d x] - 18 a^2 b d x \operatorname{Cosh}[2 (c + d x)] - 2 a^3 \operatorname{Cosh}[3 (c + d x)] + \right. \\
& 36 a^2 b c \operatorname{Log}\left[1 + \frac{a \operatorname{Sinh}[c + d x]}{b}\right] + 72 b^3 c \operatorname{Log}\left[1 + \frac{a \operatorname{Sinh}[c + d x]}{b}\right] - 36 a^2 b \\
& \left. \left(-\frac{1}{8} (2 c + \frac{i}{2} \pi + 2 d x)^2 - 4 \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i b}{a}}}{\sqrt{2}}\right] \operatorname{ArcTan}\left[\frac{(\frac{i}{2} a + b) \operatorname{Cot}\left[\frac{1}{4} (2 \frac{i}{2} c + \pi + 2 i d x)\right]}{\sqrt{a^2 + b^2}}\right] + \right. \right. \\
& \frac{1}{2} \left(2 c + \frac{i}{2} \pi + 2 d x + 4 \frac{i}{2} \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i b}{a}}}{\sqrt{2}}\right] \right) \operatorname{Log}\left[1 + \frac{(-b + \sqrt{a^2 + b^2}) e^{c+d x}}{a}\right] + \\
& \frac{1}{2} \left(2 c + \frac{i}{2} \pi + 2 d x - 4 \frac{i}{2} \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i b}{a}}}{\sqrt{2}}\right] \right) \operatorname{Log}\left[1 - \frac{(b + \sqrt{a^2 + b^2}) e^{c+d x}}{a}\right] - \\
& \frac{1}{2} \frac{i}{2} \pi \operatorname{Log}[b + a \operatorname{Sinh}[c + d x]] + \operatorname{PolyLog}\left[2, \frac{(b - \sqrt{a^2 + b^2}) e^{c+d x}}{a}\right] + \\
& \left. \operatorname{PolyLog}\left[2, \frac{(b + \sqrt{a^2 + b^2}) e^{c+d x}}{a}\right] \right) - 72 b^3 \\
& \left(-\frac{1}{8} (2 c + \frac{i}{2} \pi + 2 d x)^2 - 4 \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i b}{a}}}{\sqrt{2}}\right] \operatorname{ArcTan}\left[\frac{(\frac{i}{2} a + b) \operatorname{Cot}\left[\frac{1}{4} (2 \frac{i}{2} c + \pi + 2 i d x)\right]}{\sqrt{a^2 + b^2}}\right] + \right. \\
& \frac{1}{2} \left(2 c + \frac{i}{2} \pi + 2 d x + 4 \frac{i}{2} \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i b}{a}}}{\sqrt{2}}\right] \right) \operatorname{Log}\left[1 + \frac{(-b + \sqrt{a^2 + b^2}) e^{c+d x}}{a}\right] +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{2} \left(2 c + i \pi + 2 d x - 4 i \operatorname{ArcSin} \left[\frac{\sqrt{1 + \frac{i b}{a}}}{\sqrt{2}} \right] \right) \operatorname{Log} \left[1 - \frac{(b + \sqrt{a^2 + b^2}) e^{c+d x}}{a} \right] - \\
& \frac{1}{2} i \pi \operatorname{Log} [b + a \operatorname{Sinh}[c + d x]] + \operatorname{PolyLog} [2, \frac{(b - \sqrt{a^2 + b^2}) e^{c+d x}}{a}] + \\
& \operatorname{PolyLog} [2, \frac{(b + \sqrt{a^2 + b^2}) e^{c+d x}}{a}] + \\
& \left. 18 a (a^2 + 4 b^2) d x \operatorname{Sinh}[c + d x] + 9 a^2 b \operatorname{Sinh}[2 (c + d x)] + 6 a^3 d x \operatorname{Sinh}[3 (c + d x)] \right)
\end{aligned}$$

Problem 28: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(e + f x) \operatorname{Cosh}[c + d x]^3}{a + b \operatorname{Csch}[c + d x]} dx$$

Optimal (type 4, 400 leaves, 18 steps):

$$\begin{aligned}
& -\frac{b f x}{4 a^2 d} + \frac{b (a^2 + b^2) (e + f x)^2}{2 a^4 f} - \frac{2 f \operatorname{Cosh}[c + d x]}{3 a d^2} - \frac{b^2 f \operatorname{Cosh}[c + d x]}{a^3 d^2} - \frac{f \operatorname{Cosh}[c + d x]^3}{9 a d^2} - \\
& \frac{b (a^2 + b^2) (e + f x) \operatorname{Log} \left[1 + \frac{a e^{c+d x}}{b - \sqrt{a^2 + b^2}} \right]}{a^4 d} - \frac{b (a^2 + b^2) (e + f x) \operatorname{Log} \left[1 + \frac{a e^{c+d x}}{b + \sqrt{a^2 + b^2}} \right]}{a^4 d} - \\
& \frac{b (a^2 + b^2) f \operatorname{PolyLog} \left[2, -\frac{a e^{c+d x}}{b - \sqrt{a^2 + b^2}} \right]}{a^4 d^2} - \frac{b (a^2 + b^2) f \operatorname{PolyLog} \left[2, -\frac{a e^{c+d x}}{b + \sqrt{a^2 + b^2}} \right]}{a^4 d^2} + \\
& \frac{2 (e + f x) \operatorname{Sinh}[c + d x]}{3 a d} + \frac{b^2 (e + f x) \operatorname{Sinh}[c + d x]}{a^3 d} + \frac{b f \operatorname{Cosh}[c + d x] \operatorname{Sinh}[c + d x]}{4 a^2 d^2} + \\
& \frac{(e + f x) \operatorname{Cosh}[c + d x]^2 \operatorname{Sinh}[c + d x]}{3 a d} - \frac{b (e + f x) \operatorname{Sinh}[c + d x]^2}{2 a^2 d}
\end{aligned}$$

Result (type 4, 1315 leaves):

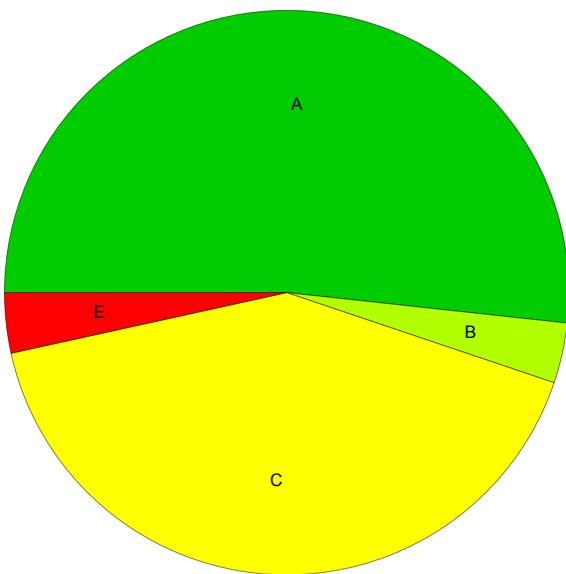
$$\frac{1}{72 a^4 d^2} \left(36 a^2 b c^2 f + 36 b^3 c^2 f + 36 i a^2 b c f \pi + 36 i b^3 c f \pi - 9 a^2 b f \pi^2 - 9 b^3 f \pi^2 + \right)$$

$$\begin{aligned}
& 72 a^2 b c d f x + 72 b^3 c d f x + 36 i a^2 b d f \pi x + 36 i b^3 d f \pi x + 36 a^2 b d^2 f x^2 + 36 b^3 d^2 f x^2 + \\
& 288 a^2 b f \operatorname{ArcSin} \left[\frac{\sqrt{1 + \frac{i b}{a}}}{\sqrt{2}} \right] \operatorname{ArcTan} \left[\frac{(i a + b) \operatorname{Cot} \left[\frac{1}{4} (2 i c + \pi + 2 i d x) \right]}{\sqrt{a^2 + b^2}} \right] + \\
& 288 b^3 f \operatorname{ArcSin} \left[\frac{\sqrt{1 + \frac{i b}{a}}}{\sqrt{2}} \right] \operatorname{ArcTan} \left[\frac{(i a + b) \operatorname{Cot} \left[\frac{1}{4} (2 i c + \pi + 2 i d x) \right]}{\sqrt{a^2 + b^2}} \right] - \\
& 54 a^3 f \operatorname{Cosh} [c + d x] - 72 a b^2 f \operatorname{Cosh} [c + d x] - 18 a^2 b d e \operatorname{Cosh} [2 (c + d x)] - \\
& 18 a^2 b d f x \operatorname{Cosh} [2 (c + d x)] - 2 a^3 f \operatorname{Cosh} [3 (c + d x)] - \\
& 72 a^2 b c f \operatorname{Log} \left[1 + \frac{(-b + \sqrt{a^2 + b^2}) e^{c+d x}}{a} \right] - 72 b^3 c f \operatorname{Log} \left[1 + \frac{(-b + \sqrt{a^2 + b^2}) e^{c+d x}}{a} \right] - \\
& 36 i a^2 b f \pi \operatorname{Log} \left[1 + \frac{(-b + \sqrt{a^2 + b^2}) e^{c+d x}}{a} \right] - 36 i b^3 f \pi \operatorname{Log} \left[1 + \frac{(-b + \sqrt{a^2 + b^2}) e^{c+d x}}{a} \right] - \\
& 72 a^2 b d f x \operatorname{Log} \left[1 + \frac{(-b + \sqrt{a^2 + b^2}) e^{c+d x}}{a} \right] - 72 b^3 d f x \operatorname{Log} \left[1 + \frac{(-b + \sqrt{a^2 + b^2}) e^{c+d x}}{a} \right] - \\
& 144 i a^2 b f \operatorname{ArcSin} \left[\frac{\sqrt{1 + \frac{i b}{a}}}{\sqrt{2}} \right] \operatorname{Log} \left[1 + \frac{(-b + \sqrt{a^2 + b^2}) e^{c+d x}}{a} \right] - \\
& 144 i b^3 f \operatorname{ArcSin} \left[\frac{\sqrt{1 + \frac{i b}{a}}}{\sqrt{2}} \right] \operatorname{Log} \left[1 + \frac{(-b + \sqrt{a^2 + b^2}) e^{c+d x}}{a} \right] - \\
& 72 a^2 b c f \operatorname{Log} \left[1 - \frac{(b + \sqrt{a^2 + b^2}) e^{c+d x}}{a} \right] - 72 b^3 c f \operatorname{Log} \left[1 - \frac{(b + \sqrt{a^2 + b^2}) e^{c+d x}}{a} \right] - \\
& 36 i a^2 b f \pi \operatorname{Log} \left[1 - \frac{(b + \sqrt{a^2 + b^2}) e^{c+d x}}{a} \right] - 36 i b^3 f \pi \operatorname{Log} \left[1 - \frac{(b + \sqrt{a^2 + b^2}) e^{c+d x}}{a} \right] - \\
& 72 a^2 b d f x \operatorname{Log} \left[1 - \frac{(b + \sqrt{a^2 + b^2}) e^{c+d x}}{a} \right] - 72 b^3 d f x \operatorname{Log} \left[1 - \frac{(b + \sqrt{a^2 + b^2}) e^{c+d x}}{a} \right] + \\
& 144 i a^2 b f \operatorname{ArcSin} \left[\frac{\sqrt{1 + \frac{i b}{a}}}{\sqrt{2}} \right] \operatorname{Log} \left[1 - \frac{(b + \sqrt{a^2 + b^2}) e^{c+d x}}{a} \right] + \\
& 144 i b^3 f \operatorname{ArcSin} \left[\frac{\sqrt{1 + \frac{i b}{a}}}{\sqrt{2}} \right] \operatorname{Log} \left[1 - \frac{(b + \sqrt{a^2 + b^2}) e^{c+d x}}{a} \right] - 72 a^2 b d e \operatorname{Log} [b + a \operatorname{Sinh} [c + d x]] - \\
& 72 b^3 d e \operatorname{Log} [b + a \operatorname{Sinh} [c + d x]] + 36 i a^2 b f \pi \operatorname{Log} [b + a \operatorname{Sinh} [c + d x]] + \\
& 36 i b^3 f \pi \operatorname{Log} [b + a \operatorname{Sinh} [c + d x]] + 72 a^2 b c f \operatorname{Log} \left[1 + \frac{a \operatorname{Sinh} [c + d x]}{b} \right] +
\end{aligned}$$

$$\begin{aligned} & 72 b^3 c f \operatorname{Log}\left[1 + \frac{a \operatorname{Sinh}[c + d x]}{b}\right] - 72 b (a^2 + b^2) f \operatorname{PolyLog}\left[2, \frac{(b - \sqrt{a^2 + b^2}) e^{c+d x}}{a}\right] - \\ & 72 b (a^2 + b^2) f \operatorname{PolyLog}\left[2, \frac{(b + \sqrt{a^2 + b^2}) e^{c+d x}}{a}\right] + 54 a^3 d e \operatorname{Sinh}[c + d x] + \\ & 72 a b^2 d e \operatorname{Sinh}[c + d x] + 54 a^3 d f x \operatorname{Sinh}[c + d x] + 72 a b^2 d f x \operatorname{Sinh}[c + d x] + \\ & 9 a^2 b f \operatorname{Sinh}\left[2 (c + d x)\right] + 6 a^3 d e \operatorname{Sinh}\left[3 (c + d x)\right] + 6 a^3 d f x \operatorname{Sinh}\left[3 (c + d x)\right] \Bigg) \end{aligned}$$

Summary of Integration Test Results

29 integration problems



A - 15 optimal antiderivatives

B - 1 more than twice size of optimal antiderivatives

C - 12 unnecessarily complex antiderivatives

D - 0 unable to integrate problems

E - 1 integration timeouts